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THE SPECTROSCOPIC SYSTEM *9 CAMELOPARDALIS*

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INTRODUCTION

The unique behavior of the sharp calcium line K as shown in radial velocity investigations of a certain class of stars was first noted by Hartmann in a discussion of the spectrum and the orbit of δ *Orionis*.[1] The large range, something over 200 km, indicated by the broad lines of this star made the comparatively constant velocity of the K line of calcium stand out prominently. Hartmann's conclusion that his range of values for K, 21 km, was due to general errors of observation has been questioned by Frost and Adams.[2] His hypothesis that the calcium line K as photographed in the spectrum of δ *Orionis* is due to a calcium cloud lying in the line of sight at some point between the solar system and this star has not been disproved and constitutes a part of the history of stellar spectroscopy.

Since that time considerable attention has been given to stars of the early spectral types which have sharp H and K lines, and a list of objects known to belong to this class would at the present time include a hundred or more stars. From statistical evidence Frost has proposed the presence of sharp H and K lines with the

[1] *Astrophysical Journal*, **19**, 268, 1904.

[2] *Ibid.*, **19**, 351, 1904.

other lines in the spectrum broad and diffuse as criterion that the star is probably binary.[1]

A number of stars of this class have been shown to give anomalous results for the calcium lines when these are compared with results from the other lines. Three detailed investigations only besides that by Hartmann have been published on this type of star. Jordan[2] has derived the orbits of both components of *o Persei*. He measured the K line on 60, and the H line on 22 of the 70 plates discussed in his paper. Considerations on the motion of the center of gravity of the system and the mean velocity shown by the calcium lines "seem to indicate that this line probably has its origin in the system." Daniel and Schlesinger[3] in discussing the calcium line problem in connection with an orbit determination of *β Scorpii* admit the possibility of a small oscillation with the same period as that of the orbital revolution. Duncan[4] in discussing the orbit of *β Scorpii* from plates obtained by Slipher finds that the constant calcium velocity differs by 10 km from that of the center of gravity of the system. He concludes that this evidence supports the hypothesis of a detached calcium cloud in the line of sight.

Shortly after the appearance of Hartmann's paper Frost and Adams announced that measures on the broad lines of the star *9 Camelopardalis* showed it to have variable radial velocity.[5] Assured by the character of the two calcium lines H and K and also by the good agreement of their measurements upon them, these observers asserted that the two lines actually vary, the variation differing in phase and magnitude from that shown by the broad lines. Their original values as published are given for convenience in Table I. The calcium values are attributed, probably as a convenience, to the second component of the system. Frost and Adams interpreted the variation of these lines as real, as distinct from the variation in the broad lines, and placed its source in the star system itself. Since this announcement was made a small

[1] *Astrophysical Journal*, **29**, 234, 1909.

[2] *Publications of the Allegheny Observatory*, **2**, 63, 1910.

[3] *Ibid.*, **2**, 127, 1912.

[4] *Publications of the Astronomical and Astrophysical Society of America*, **2**, 82, 1912.

[5] *Astrophysical Journal*, **19**, 350, 1904.

number of stars giving similar results has been found, such as *37 φ′ Orionis*, *ξ Persei*,[1] and *ν Geminorum*.[2] Since this paper was completed Cannon[3] has published an orbit of *ξ Persei* from measures on H and K.

TABLE I

Plate	Date	G.M.T.	Taken by	Velocity		No. of Lines		Velocity Mean
				Frost	Adams	Frost	Adams	
				km	km			km
I B 194....	1903 Nov. 17...	$15^h\ 9^m$	A	+14	+ 6	5	3	+10
229....	Dec. 26...	21 59	F	+ 3	− 2	3	4	+ 1
254....	1904 Jan. 2...			+10	+14	2	3	+12
	Second comp.	21 30	A	− 9	− 6	2	2	− 8
285....	Feb. 26...			+ 9	+13	6	4	+11
	Second comp.	17 10	A	+ 2	+ 3	2	2	+ 3
288....	Mar. 8...			− 6	− 7	3	5	− 7
	Second comp.	17 21	F	+ 8	+ 6	2	2	+ 7
296....	Mar. 19...			− 1	+ 4	3	4	+ 2
	Second comp.	16 2	A	−12	−12	2	2	−12

Several years ago Professor Frost suggested to the writer the desirability of an investigation of the anomalous calcium type of spectroscopic binary and the present paper is the outcome of such an investigation. Evidently where the variation to be inquired into is as small as it is in all stars of this type, it is important to choose one in the spectrum of which both the H and K lines are strong and sharp and therefore measurable with some degree of accuracy. The star *9 Camelopardalis* fulfils this requirement. The H_ϵ line of hydrogen is so weak that it interferes in no way with the H line of calcium. Another consideration which favors the choice of this star is its position in the sky. Its declination is +60°, which enables the observer to follow it comfortably for 12 hours on a single night, if necessary, and its right ascension is 5^h, which brings the star into opposition about December 1, when the nights are at their longest. Unfortunately, the broad lines do not give a large range of velocities. However, since the calcium anomaly led to this investigation it was early determined that the advantages given above outweighed this disadvantage.

[1] *Ibid.*, **30**, 62, 1909.

[2] *Ibid.*, **32**, 300, 1910.

[3] *Journal of the Royal Astronomical Society of Canada*, **6**, 188, 1912.

SPECTRUM OF 9 *Camelopardalis*

($\alpha = 4^h\ 44^m.1$; $\delta = +66°\ 10'$; Mag. = 4.34; H.R. Type B)

Vogel and Wilsing[1] give a list of nineteen lines which they were able to measure and identify in this spectrum. Table II shows what lines are strong enough to be readily seen on our Bruce plates. The lines with asterisks alone were used in obtaining radial velocities, and the lines which have double asterisks were given most weight.

TABLE II

Element	λ		Element	λ	
H+*He*.....	3889.5	$H\zeta$	*He*.....	4143.92*	
Ca.....	3933.82	K	*H*.....	4340.63**	$H\gamma$
Ca.....	3968.62	H	*He*.....	4388.10*	
H.....	3970.21*	$H\epsilon$	*He*.....	4471.65**	
He.....	4026.34*		*C*.....	4647.6	Lockyer
Si.....	4089.10**		*C*.....	4650.8	Lockyer
N.....	4097.47*	Lockyer	*He*.....	4713.25	
H.....	4101.90	$H\delta$	*H*.....	4861.63*	$H\beta$
Si.....	4116.40**		*He*.....	4922.10	

An idea of the appearance of the spectrum may be obtained from the enlarged spectrogram shown in Plate I. More will be said about the individual lines later in the discussion.

THE OBSERVATIONS

Plates from the first up to and including No. 2671 were made with the Seed "non-halation" double-coat emulsion; all the remaining plates but one were taken with the "Seed 30." For No. 2969 the fine-grained "Seed 23" was used. All the plates used in this series were taken with the usual one-prism arrangement of the Bruce spectrograph. In order to make the calcium lines stand out sharply, and to narrow the broad lines down to the cores, the majority of the plates were strongly overexposed. The average exposure time was 88 minutes. Table III gives all the necessary data for the plates used in this paper. Unless both calcium lines could be well measured the plate has been omitted. Under the column "Taken by" are placed the initials of the observer by whom

[1] *Pub. Astrophys. Observatoriums zu Potsdam*, **12**, 34, 1902.

TABLE III

Plate	Taken by	Date	G.M.T.	Julian Day	Velocity *Ca* Lines	Wt.	Velocity Broad Lines	No. of Lines
I B 194.....	A	1903 Nov. 17	15h 9m	2416436.63	− 2	1	+ 6	5
229.....	F	Dec. 26	21 59	6475.92	− 4	2	0	4
254.....	A	1904 Jan. 2	21 30	6482.90	−10	4	+ 9	3
285.....	A	Feb. 26	17 10	6537.72	− 4	5	+ 8	5
288.....	F	Mar. 8	17 21	6548.72	+ 4	3	+16	3
296.....	A	Mar. 19	16 2	6559.67	−14	4	+ 1	6
654.....	B	1905 Dec. 25	21 49	7205.91	+ 1	2	+ 4	5
663.....	B	1906 Jan. 26	20 10	7237.84	− 2	4	+ 6	6
867.....	B	Sept. 24	21 24	7478.89	−10	1	− 2	5
893.....	F B	Oct. 31	18 46	7515.78	+ 1	5	− 7	5
1159.....	B	1907 Sept. 2	21 52	7821.91	.....	..	+ 9	6
1181.....	L	Sept. 23	22 25	7842.93	− 5	4	+13	6
1189.....	L	Sept. 24	22 12	7843.92	.....	..	− 1	5
1200.....	F	Oct. 11	20 34	7860.86	− 4	4	+27	5
1210.....	B	Oct. 18	19 56	7867.83	− 4	4	+26	6
1234.....	P	Oct. 22	19 23	7871.81	− 6	3	−11	6
1241.....	B	Nov. 22	17 19	7902.72	−10	4	− 7	5
1245.....	L	Nov. 23	15 42	7903.65	.....	..	0	7
1256.....	L	Nov. 27	21 55	7907.91	.....	..	+18	6
1261.....	B	Nov. 30	17 39	7910.74	− 3	3	+18	6
1812.....	L	1908 Oct. 30	22 46	8245.88	− 7	2	− 3	6
1821.....	B	Nov. 2	20 27	8248.85	+ 7	1	− 5	6
1850.....	L	Nov. 10	22 58	8256.96	+ 4	3	− 1	6
2113.....	F L	1909 Aug. 23	21 28	8542.89	−11	4	− 2	7
2134.....	L	Sept. 10	22 9	8560.92	.....	..	− 9	5
2170.....	L	Oct. 29	22 48	8609.95	− 6	1	−14	8
2203.....	L	Nov. 29	15 28	8640.64	.....	..	+ 4	10
2278.....	F	1910 Feb. 7	14 49	8710.62	− 3	1	+ 2	6
2611.....	L	Dec. 29	15 46	9035.66	+ 3	4	− 9	8
2671.....	B Ar	1911 Feb. 10	12 55	9078.54	0	1	+10	9
2808.....	B	Aug. 25	21 46	9274.91	+ 3	2	+44	6
2814.....	Ar	Aug. 28	21 23	9277.89	− 5	1	+14	6
2831.....	L	Sept. 15	21 37	9295.90	+10	3	+12	5
2836.....	B	Sept. 18	22 00	9298.92	− 2	5	+32	6
2839.....	B	Oct. 23	19 1	9333.79	+ 5	5	+ 8	5
2843.....	F	Oct. 31	18 29	9341.77	− 6	5	+37	5
2846.....	F	Nov. 7	15 30	9348.65	+ 6	4	+ 4	5
2850.....	L	Nov. 7	22 18	9348.93	0	3	− 3	7
2857.....	L	Nov. 18	15 9	9359.63	+ 3	4	+ 9	9
2867.....	F	Nov. 25	21 40	9366.90	0	5	+18	8
2871.....	L	Nov. 26	14 33	9367.61	+11	4	+ 7	8
2875.....	B	Nov. 26	21 17	9367.89	− 3	4	+10	9
2880.....	B	Nov. 30	15 29	9371.65	− 1	4	+16	6
2884.....	F L	Dec. 3	14 48	9374.62	− 2	5	+15	8
2896.....	L	Dec. 4	21 46	9375.91	+ 4	5	+11	7
2902.....	F B	Dec. 12	16 16	9383.68	+ 3	4	+10	6
2905.....	L	Dec. 12	21 9	9383.88	0	4	+ 9	9
2906.....	L	Dec. 12	22 46	9383.95	+ 7	4	+ 6	7
2918.....	L	Dec. 24	12 1	9395.50	+ 2	1	+ 8	7
2919.....	L	Dec. 24	14 7	9395.59	− 2	2	+ 6	7
2929.....	B L	1912 Jan. 2	12 20	9404.51	+ 7	2	+12	5
2930.....	L B	Jan. 2	14 5	9404.59	+ 5	4	+15	4
2932.....	B	Jan. 2	16 48	9404.75	− 1	1	+ 4	3

TABLE III—*Continued*

Plate	Taken by	Date	G.M.T.	Julian Day	Velocity *Ca* Lines	Wt.	Velocity Broad Lines	No. of Lines
I B 2933.....	B L	1912 Jan. 2	18h34m	2419404.78	+ 7	2	+ 3	3
2934.....	L	Jan. 2	20 40	9404.86	+ 6	5	+14	4
2938.....	L B	Jan. 6	12 48	9408.53	− 3	3	+ 7	7
2939.....	L	Jan. 6	14 42	9408.61	− 4	5	+11	5
2940.....	L	Jan. 6	16 29	9408.69	− 4	3	+ 6	6
2941.....	L B	Jan. 6	18 10	9408.76	− 7	2	+ 2	7
2942.....	B	Jan. 6	20 06	9408.84	− 4	2	− 7	4
2946.....	B F	Jan. 9	12 12	9411.51	− 3	3	+10	8
2947.....	F	Jan. 9	13 58	9411.58	− 7	4	+ 5	5
2949.....	F L	Jan. 9	17 39	9411.74	− 2	4	+11	5
2953.....	L B	Jan. 11	18 29	9413.77	− 6	5	+ 1	5
2954.....	B	Jan. 11	20 30	9413.86	− 6	4	− 6	8
2957.....	B L	Jan. 15	12 08	9417.50	− 3	5	+18	4
2958.....	B L	Jan. 15	14 00	9417.58	+ 4	2	−26	6
2959.....	B	Jan. 15	15 50	9417.66	− 4	4	− 4	5
2960.....	B L	Jan. 15	17 26	9417.73	− 5	4	− 9	4
2961.....	L	Jan. 15	18 56	9417.79	− 2	5	− 3	6
2962.....	L	Jan. 15	20 40	9417.86	− 3	3	− 2	7
2965.....	B	Jan. 23	12 18	9425.51	+ 4	4	+14	6
2966.....	B	Jan. 23	14 3	9425.58	+ 2	5	+19	5
2967.....	B	Jan. 23	16 5	9425.67	+ 3	4	+17	5
2969.....	B L	Jan. 23	19 41	9425.82	− 3	5	− 1	5
2981.....	B	Feb. 8	12 48	9441.53	+ 1	2	− 3	4
2983.....	B	Feb. 8	16 48	9441.70	− 3	3	0	5

the plate was secured: A=Adams; Ar=Arbogast; B=Barrett; F=Frost; L=Lee; P=Parkhurst. The efficient and friendly assistance of Mr. F. R. Sullivan, engineer in charge of the telescope, is gratefully acknowledged.

The calcium lines on the plates have been measured twice and in some cases three times. These measurements were made so far apart that no prejudice of memory would prevent them from being independent. The weights given were derived in the following manner: (*a*) All the plates were examined consecutively without referring to their measures and the weight 0 to 5 assigned to them based upon the completeness of exposure, spark and other instrumental adjustments, freedom of the calcium lines from grain distortion, etc. (*b*) The measures were then examined and from them each plate was again weighted as follows:

For H and K within 2 km and means of double measures within 1 km, weight = 5

For H and K within 3 km and means of double measures within 1.5 km, weight = 4

For H and K within 4 km and means of double measures within 2 km, weight = 3
For H and K within 6 km and means of double measures within 2 km, weight = 2
For H and K within 7 km and means of double measures within 2.5 km, weight = 1
For H and K more than 10 km apart, regardless of agreement of means, weight = 0

Now the weight given for each observation is the result of combining its (*a*) and (*b*) in the ratio of $\frac{2}{5}$ and $\frac{3}{5}$ respectively. For an illustration, consider the measures of Plate No. 663, Table IV. The first measure of H and K yielded −1.4 and −3.0 km respectively for the two lines; the second measure gave in the same way −2.7 and −0.9 km. In each pair the agreement is within 2 km, and the means −2.2 and −1.8 agree far within 1 km. Hence the (*b*) weight of this plate is 5. The appearance of the plate and of the lines was, however, not so reassuring, so that the (*a*) weight 3 was given to it. The final weight of the plate was rounded off to 4. Since the two lines are in general of the same character in this star, this procedure is obviously justifiable. This mode of weighting the observation from each plate was used in preference to that of taking the sum of the line weights for the weight of the observation. Line weights will relate properly the different values from one plate, but unless the observer completes a series of plates of one star without measuring other types of spectra in between, he will hardly be able to keep his standards of weighting constant. Obviously this must be done to obtain a fair comparison of plate values from line weights. It is needless to say that weights are arbitrary and in observations of this kind do nothing more than indicate a probability that one observation is better than another.

Table IV contains the individual measures of the H and K lines. For the sake of keeping the series as uniform as possible only the writer's measures of the Yerkes Observatory plates have been used. All the measures with a few exceptions were made and the weights assigned before any of the results hereafter described were derived. Plates of this star with measures were kindly sent from Allegheny and Ottawa upon request. I remeasured these,

TABLE IV

Plate	First Measure			Second Measure			Mean Velocity
	K	H	Velocity$_1$	K	H	Velocity$_2$	
I B 194	0.0	+ 0.1	0.0	− 5.9	− 1.1	− 3.5	− 1.8
229	− 9.7	− 0.8	− 5.2	− 4.5	− 2.0	− 3.2	− 4.2
254	−11.2	−10.7	−10.9	−11.6	− 6.4	− 8.5	− 9.7
285	− 4.7	− 3.4	− 4.0	− 2.6	− 5.9	− 3.2	− 3.6
288	+ 4.7	+ 2.1	+ 3.4	+ 6.1	+ 1.8	+ 4.2	+ 3.8
296	−16.7	−12.0	−14.4	−14.3	−11.5	−12.9	−13.6
654	+ 0.9	− 0.8	0.0	+ 1.4	+ 3.0	+ 2.2	+ 1.1
663	− 1.4	− 3.0	− 2.2	− 2.7	− 0.9	− 1.8	− 2.0
867	− 6.9	−12.5	− 8.8	− 3.6	−18.3	−10.9	− 9.8
893	− 1.7	+ 1.5	− 0.1	+ 2.4	+ 2.3	+ 2.3	+ 1.1
1181	− 4.7	− 4.4	− 4.6	− 7.2	− 5.0	− 6.1	− 5.4
1200	− 1.0	− 6.8	− 3.9	− 2.0	− 6.5	− 4.2	− 4.0
1210	− 2.4	− 3.0	− 2.7	− 6.3	− 3.9	− 5.1	− 3.9
1234	− 1.4	− 9.4	− 5.4	− 8.9	− 5.7	− 7.3	− 6.4
1241	− 8.5	− 9.0	− 8.8	−11.6	−10.1	−10.9	− 9.8
1261	0.0	− 6.5	− 2.6	− 2.9	− 4.8	− 4.2	− 3.4
1812	− 8.3	− 1.7	− 5.7	− 7.9	− 8.2	− 8.1	− 6.9
1821	+14.3	+ 5.9	+ 8.0	+10.0	+ 3.3	+ 6.7	+ 7.4
1850	+ 4.0	+ 5.6	+ 4.8	+ 3.1	+ 5.2	+ 4.1	+ 4.4
2113	−10.6	−11.6	−11.1	− 9.8	−12.0	−10.9	−11.0
2170	−10.3	− 1.3	− 6.7	−10.5	+ 1.3	− 4.6	− 5.6
2278	− 6.3	+ 1.1	− 3.3	− 8.5	+ 1.1	− 3.7	− 3.5
2611	+ 1.5	+ 6.3	+ 3.9	+ 1.2	+ 4.6	+ 2.9	+ 3.4
2671	− 5.5	+ 3.7	− 0.9	− 0.3	+ 3.3	+ 1.5	+ 0.3
2808	+ 0.5	+ 4.2	+ 2.1	+ 0.1	+ 5.6	+ 3.8	+ 3.0
2814	−12.3	− 3.3	− 6.9	− 5.4	− 1.6	− 3.5	− 5.2
2831	+11.2	+11.0	+11.1	+ 8.1	+ 9.4	+ 9.2	+10.2
2836	− 2.5	− 0.6	− 1.5	− 2.5	− 0.7	− 1.9	− 1.7
2839	+ 4.9	+ 5.4	+ 5.1	+ 4.7	+ 4.7	+ 4.7	+ 4.9
2843	− 6.6	− 3.8	− 5.2	− 7.1	− 4.6	− 5.8	− 5.5
2846	+ 3.2	+ 9.0	+ 5.4	+ 4.8	+ 9.1	+ 6.2	+ 5.8
2850	+ 0.2	− 1.3	− 0.8	+ 0.8	− 0.1	+ 0.4	− 0.2
2857	+ 1.9	+ 4.9	+ 3.4	+ 0.9	+ 5.0	+ 3.0	+ 3.2
2867	− 0.4	− 1.4	− 0.8	− 0.4	0.0	− 0.2	− 0.5
2871	+ 9.3	+15.6	+10.9	+10.3	+10.6	+10.4	+10.6
2875	− 3.2	− 3.5	− 3.3	− 4.2	− 1.3	− 3.4	− 3.4
2880	− 3.8	+ 2.3	− 1.2	− 4.3	+ 1.8	− 1.7	− 1.4
2884	− 4.0	+ 1.7	− 2.6	− 3.2	− 0.7	− 2.4	− 2.5
2896	+ 3.6	+ 5.1	+ 4.2	+ 3.9	+ 4.4	+ 4.2	+ 4.2
2902	+ 1.2	+ 3.8	+ 2.5	+ 1.2	+ 6.2	+ 3.7	+ 3.1
2905	− 2.4	+ 2.9	+ 0.2	− 2.4	+ 3.2	+ 0.4	+ 0.3
2906	+ 6.7	+ 6.0	+ 6.3	+ 8.3	+ 6.8	+ 7.4	+ 6.8
2918	− 1.1	+ 9.0	+ 2.9	− 3.1	+ 6.1	+ 0.6	+ 1.8
2919	− 3.4	+ 3.9	− 1.0	− 4.2	+ 1.3	− 2.0	− 1.5
2929	+ 7.7	+ 1.3	+ 5.1	+ 9.5	+ 8.8	+ 9.2	+ 7.2
2930	+ 5.1	+ 7.2	+ 6.0	+ 3.6	+ 6.3	+ 4.8	+ 5.4
2932	− 7.4	+ 3.2	− 0.3	− 8.5	+ 3.1	− 0.8	− 0.6
2933	+ 3.1	+ 8.6	+ 7.2	+ 2.6	+ 8.7	+ 6.7	+ 7.0
2934	+ 4.3	+ 6.9	+ 5.6	+ 4.9	+ 7.3	+ 6.2	+ 5.9
2938	− 5.1	+ 0.6	− 3.0	− 6.3	+ 0.7	− 3.3	− 3.2
2939	− 3.7	− 5.1	− 4.4	− 5.2	− 4.4	− 3.7	− 4.0
2940	− 7.6	− 0.3	− 3.9	− 7.1	− 2.0	− 5.1	− 4.5

TABLE IV—*Continued*

Plate	First Measure			Second Measure			Mean Velocity
	K	H	Velocity$_2$	K	H	Velocity$_2$	
I B 2941	− 9.9	− 5.0	− 8.0	− 9.0	− 0.2	− 6.2	− 7.1
2942	− 7.8	− 1.6	− 4.3	− 9.7	− 0.6	− 4.5	− 4.4
2946	− 4.4	+ 0.8	− 1.8	− 7.4	+ 0.8	− 3.3	− 2.6
2947	− 6.2	− 8.0	− 7.3	− 6.5	− 9.2	− 7.6	− 7.4
2949	− 4.4	+ 1.4	− 2.5	− 4.6	+ 1.5	− 2.3	− 2.4
2953	− 5.1	− 6.8	− 6.0	− 5.1	− 7.2	− 6.2	− 6.1
2954	− 8.3	− 4.8	− 7.0	− 8.3	− 1.5	− 6.0	− 6.5
2957	− 3.7	− 2.1	− 2.9	− 1.5	− 4.1	− 2.7	− 2.8
2958	− 0.9	+ 8.0	+ 2.7	+ 1.4	+ 8.6	+ 4.3	+ 3.5
2959	− 5.6	− 2.2	− 4.1	− 5.2	− 1.1	− 3.4	− 3.8
2960	− 5.9	− 6.6	− 6.2	− 5.6	− 2.5	− 4.4	− 5.3
2961	− 0.4	− 2.9	− 1.7	− 0.4	− 4.0	− 1.9	− 1.8
2962	− 5.9	+ 0.2	− 3.9	− 2.6	+ 1.1	− 1.4	− 2.6
2965	+ 4.4	+ 4.8	+ 4.6	+ 6.0	+ 3.0	+ 4.3	+ 4.4
2966	− 0.8	+ 2.9	+ 1.3	+ 0.5	+ 5.0	+ 3.1	+ 2.2
2967	+ 3.8	+ 0.4	+ 1.8	+ 3.8	+ 4.3	+ 4.1	+ 3.0
2969	− 4.0	− 2.1	− 3.2	− 1.3	− 2.4	− 1.9	− 2.6
2981	− 2.2	+ 4.7	+ 1.9	− 2.6	+ 3.7	+ 1.0	+ 1.4
2983	− 5.7	− 2.0	− 3.8	− 5.7	+ 0.5	− 2.6	− 3.2

but the necessity of having uniform material soon became apparent so that they were not utilized. All the reductions have been made with the aid of tables constructed by the writer in the manner proposed by Schlesinger.[1]

To see if the relatively very sharp H and K lines might nevertheless show double lines, a careful examination of them was made on the plates. The conclusions were: (*a*) The calcium lines are simple in this star; (*b*) a considerable difference in the width of the lines can be accounted for as a function of fulness of exposure, development, and accidental grain structure of the plate.

The labor involved in deriving the period of variation of the calcium lines has been considerable. At first, simple variations from the sine curve were tried, and then various curves derivable from elliptic motion. The period was not found until 45 or more probable periods, ranging from 0.4 to 20 days in length, had been tested. Where the range of oscillation is so small, single observations are often misleading, and therefore it was necessary to ascertain that the greater part of the observations were not satisfied before rejecting the period. The adopted value of the period of

[1] *Publications of the Allegheny Observatory*, **1**, 9, 1910.

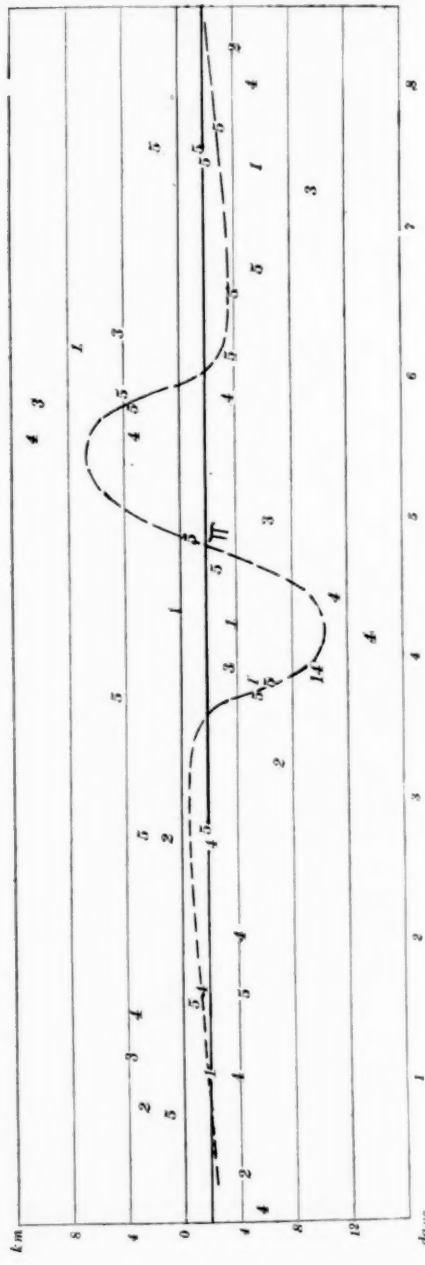

FIG. 1.—Velocity-curve of 9 *Cameloparda lis* .

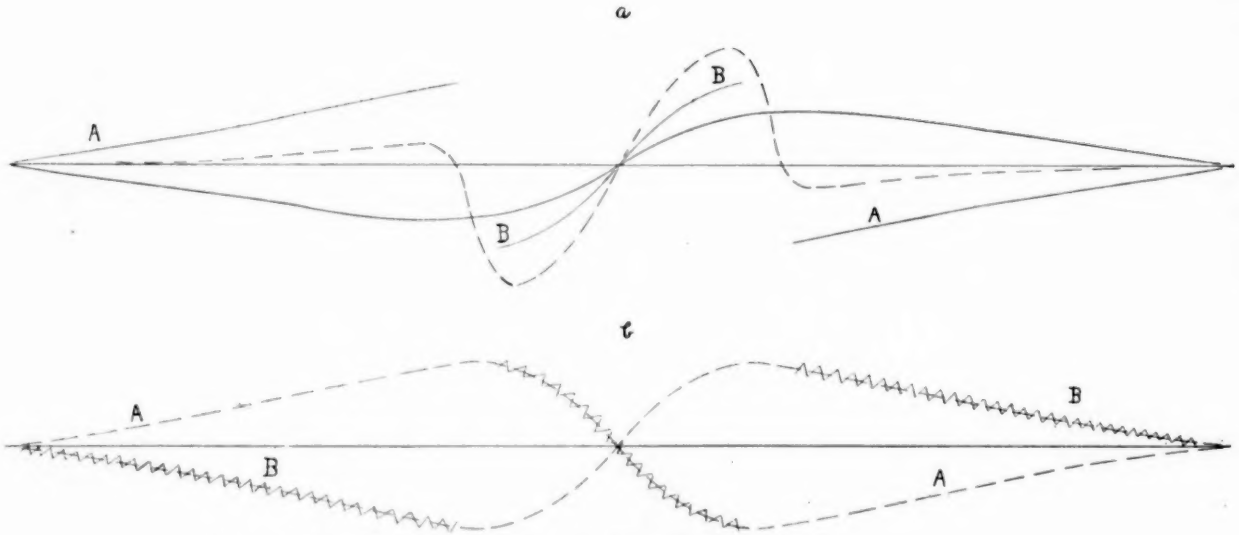

FIG. 2.—*a*: Smooth curve is velocity-curve of primary
AA and BB are uneclipsed portions of velocity-curve of the calcium masses
Dash-curve is algebraic sum of the other curves
b: Velocity-curves of calcium masses, crossed curve representing eclipsed portions

variation is 7.9957 days. Since observations extend over an interval of about 3000 days, or 375 complete periods, the length of the period is defined rather precisely. A change of one unit in the last place is sufficient to distort the curve appreciably. In the velocity-curve (Fig. 1) it has seemed desirable to give each observation used rather than the normal point with respect to phase. Also, instead of giving a dot or circle for each observation, a number has been used which indicates its weight. When several plates were taken in one night the mean of all or of a group is given as an observation.

Let the γ axis be drawn parallel to the time axis, and so adjusted that the areas above and below it shall be equal. For all practical purposes the curve is now symmetrical about the point π. Obviously the velocity-curve is not even approximately that of one component of a binary. It should be borne in mind that it would be as difficult to explain this velocity-curve by assuming a two-body system merely, were the parts of the curve before the low minimum and after the high maximum to approach closely to the γ axis, as it is when they actually cross it. Hence a small error in tracing the curve leaves the problem essentially as it is. While the assumption that the system is triple or quadruple might explain the form of the velocity-curve, it would not account for the sharp calcium-line anomaly without identifying the source of these lines with one of the bodies of the system. Thus it will be necessary to assume a physical situation which not only produces the observed velocity-curve, but which also takes account of the difference in the appearance of the H and K lines and the other lines in the spectrum.

In the development which follows, the reader will bear in mind that the indicative mood is used merely as a convenience.

Fig. 3 gives an outline of the system which is assumed. It is a two-body system with certain added elements. The symmetry of the chief minimum with the chief maximum portion of the curve suggests that the axis of the elliptical orbit lies approximately in the line of sight. The periastron point in the orbit of the primary lies nearest to the observer, and the inclination is taken to be nearly 90°. The secondary body is small as compared with the primary,

and its orbit has not been drawn. The center of gravity of the system lies close to the surface of the primary. The primary is a star of an early spectral type, the atmosphere of which contains large areas that have rapidly changing density-gradients. The calcium vapor here, as in the sun, lies at a greater distance from the generating body than the vapors of the other elements.[1] While this outer envelope is continuous about the two components of the binary, the spectrographically effective part of it are the appendages shown in Fig. 3.

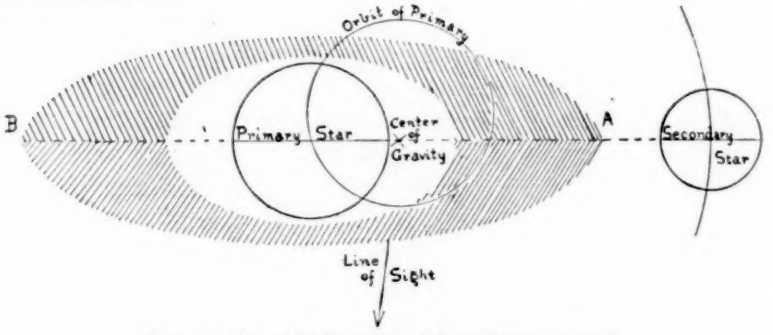

FIG. 3.—Equatorial section of the system assumed

On account of the greater distance of these vapor masses from the primary, the material in them is in a relatively quiescent state, and since it is not subject to the radical changes in vapor-pressure and temperature which agitate the lower-lying sources of the other lines in the spectrum, it gives the narrow H and K lines observed.

[1] Charles E. St. John (*Astrophysical Journal*, **32**, 80, 1910) speaking of chromospheric calcium says: "The high velocity of the calcium vapor producing the K_3 line points to a higher elevation of this layer of calcium vapor than of the hydrogen effective in the production of the H_α line."

Sir Norman Lockyer, in reporting observations of the eclipse of 1898, gives these chromospheric heights of the various chemical elements (quoted in Abbot, *The Sun*, p. 171): *Ca* (K), 9700 km; Hydrogen, 7200 km; Helium, 5400 km; *Ca* (4227), 3200 km; *Fe*, 2300 to 100 km for iron lines of different classes.

See Abbot, *The Sun*, pp. 155–172, for a suggestive description of cloud formations in the solar atmosphere.

Hale and Ellerman, using the K line of calcium, photographed a prominence to a height of 452,000 km (*Astrophysical Journal*, **1**, 434, 1895). Fox, using the H line, obtained a photograph of an eruptive prominence which reached 316,900 km (*ibid.*, **26**, 155, 1907). Slocum photographed another which reached 319,500 km, before running off the edge of the plate (*ibid.*, **32**, 128, 1910).

The secondary star tends to draw this high-level calcium into two regions, one on each side of the primary. Possibly the centers of these regions coincide with two of the three points of zero relative velocity which lie in the line joining the two stars. Hence the secondary star controls the positions of these masses as the orbital velocities of the two stars vary. That is to say, the varying angular velocity of revolution of the calcium clouds is not proportional to the angular velocity of rotation of the primary on its axis, which probably takes place once for every revolution in its orbit, but it is proportional to the velocity of the primary in its orbit. The motion of the calcium vapor is not of the nature of a tidal wave which would in general give no radial velocity effect, but it is a motion of translation of the cloud of vapor. The calcium masses exist as bodies practically separate from the star, and the friction between them and the lower strata of the stellar atmosphere is insignificant. Therefore motion of translation with respect to the surface of the primary is possible.

Fig. 4 shows the primary and the spectrographically effective centers of the calcium masses in critical positions, but all reduced in size relative to the orbit so as to avoid making the figure confusing to the eye. Obviously the secondary star is found at all times at the end of a line drawn from the center of the primary through the center of gravity, and terminating at the outer orbit. From the motion of the primary in its elliptical orbit, the revolution of the masses about the primary, and their alternate eclipse by it, it is proposed to duplicate the observed velocity-curve. In order to show this in a fairly accurate way, use has been made of the principle of the hodograph which may be stated as follows: the path being an ellipse described about the focus, S, under the law of the inverse square, the hodograph is the auxiliary circle, the other focus, H, being the origin, and HQ drawn perpendicular to the tangent at P, where P is any point on the ellipse, and Q the corresponding point on the hodograph. Fig. 5 is double: the part below the line π 10 is half of an ellipse of eccentricity 0.30, in which adjacent focal radii are drawn so as to include equal areas. A few of these near the periastron point have been divided into halves. The part of the figure above the π 10 line is half of the auxiliary

circle and the lengths of its radii drawn as stated above are proportional to the velocities in the ellipse. The data derived from the hodograph and their transformations for use in the radial velocity-curves are given in Table V.

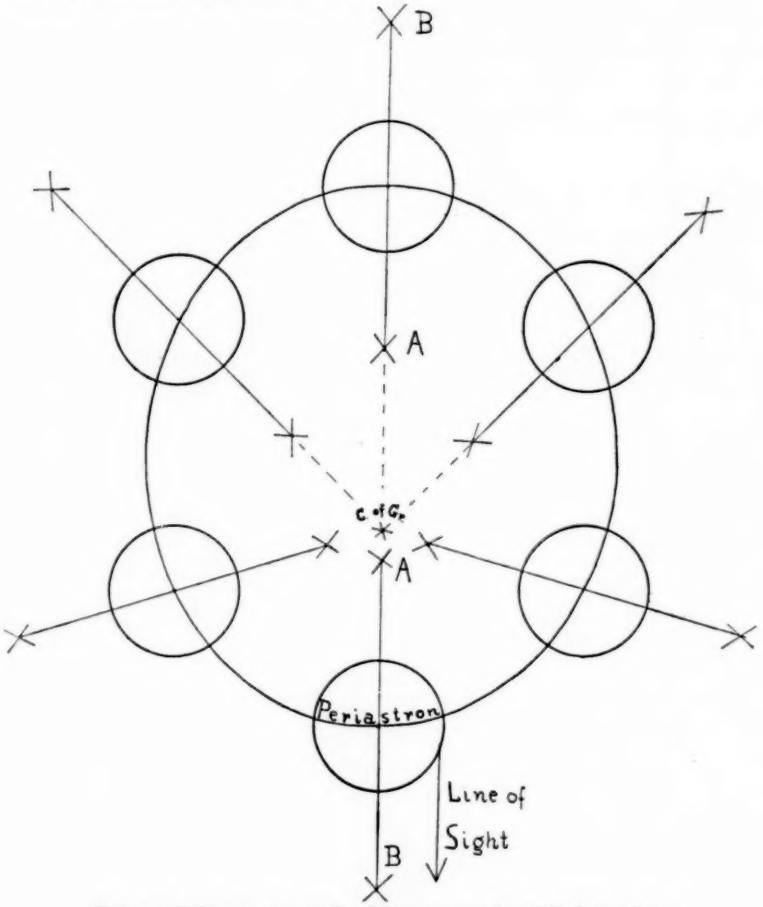

FIG. 4.—Primary star and calcium masses in critical positions

The orbital motion of the primary is always tangential to the ellipse and the projections upon the line of sight are given in Table V under (4). The velocities of the calcium masses are derived regarding the center of the primary as a stationary point. By our assumption on p. 14 their angular velocities also are

proportional to the numbers in (3), Table V. This element of motion is directed, not tangentially to the ellipse, but perpendicularly to the radius vector, and the projections upon the line of sight given in (6), Table V, are taken accordingly. The proportionality factors by which the quantities in (4) and (6) are converted to those in (5) and (7), respectively, not only fix the scale

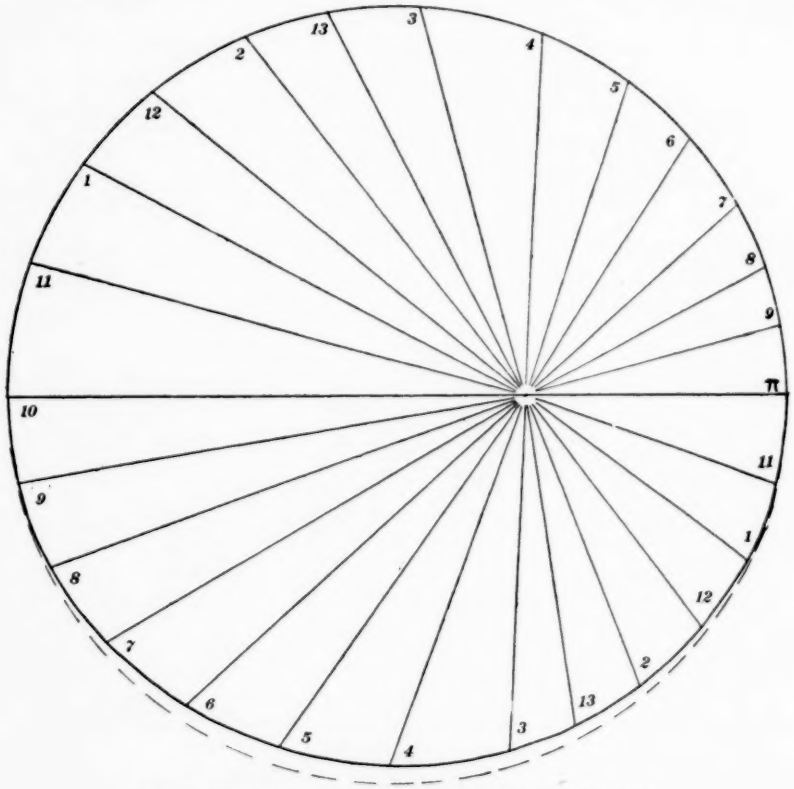

FIG. 5.—One-half of orbital ellipse and hodographic circle

of the curves in Fig. 2, but also determine to a degree the relative scale in the system which has been assumed. Obviously in the case under consideration, these factors are arbitrary within certain limits. In Fig. 2*a*, plotted from (5), Table V, is shown as a smooth line the radial velocity-curve of the orbital motion of the primary alone. In Fig. 2*b*, drawn from data in (7), Table V, are shown the

radial components of the velocities of the calcium masses relative to the center of the primary star regarded as a stationary point. The crossed portions of these curves represent the parts of the orbits of the calcium masses described about the primary during eclipse by the primary. The uncrossed parts in *b*, drawn as *AA* and *BB* in *a*, are corrections to be applied to the smooth curve in *a*. The algebraic sum of these is the dash-curve in *a*, which is, as will readily be seen, a close duplicate of the observed

TABLE V

	Orbital Motion of Primary				Motion of Calcium Masses	
No.	Time from Periastron	Nos. Proportional to Velocity in Ellipse	Projection upon Line of Sight	(4)×$\frac{1}{14}$	Velocity about Center of Primary	(6)×$\frac{1}{10}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Days					
π	0.0	131	0	0.0	0	0.0
11	0.2	130	34	2.4	43	4.3
1	0.4	127	60	4.3	76	7.6
12	0.6	122	78	5.6	97	9.7
2	0.8	116	92	6.6	108	10.8
13	1.0	110	97	6.9	109	10.9
3	1.2	102	99	7.1	100	10.0
4	1.6	92	92	6.6	85	8.5
5	2.0	84	79	5.6	69	6.9
6	2.4	78	65	4.6	52	5.2
7	2.8	72	48	3.4	37	3.7
8	3.2	69	32	2.3	22	2.2
9	3.6	67	17	1.2	10	1.0
10	4.0	66	0	0.0	0	0.0

velocity-curve given in Fig. 1. The parts of the composed curve which represent the shifting of the spectroscopically predominant factor from *A* to *B* and from *B* to *A*, Fig. 2*b*, by the eclipse operation cannot be definitely drawn. At these points, corresponding to the steep drops in the curve *to* the primary minimum and *from* the primary maximum, the peculiarities of the physical situation show to greatest effect, and the correction to the velocity-curve of the star is for a time indeterminate. This uncertainty is also shown in the observed velocity-curve by the

observational values at these points. On considering the complexity of the physical situation, the velocities at these times could hardly be expected to follow a curve very closely.

Four of the orbital elements, P, K, μ, and γ, come directly from the observations and from the velocity-curve; the other four elements arise from the assumed orbit. The orbital elements are

$P = 7.9957$ days
$e = 0.30$
$\mu = 45^{\circ}.024$
$K = 9.0$ km
$\omega = 90^{\circ}$ or 270° closely
$T =$ J.D. 2416480.35
$\gamma = -2.25$ km
$i = 90^{\circ}$ closely

Reasonable physical and orbital differences in stars of this class easily give rise to a number of anomalies of calcium such as have been observed. Consider a binary system in which the two bodies are comparable in light, and hence probably of approximately the same mass. When the time of rotation and the period of revolution are equal, the mutually facing sides of the two bodies are much hotter than the sides facing away, and the incandescent calcium collected from both stars into some point between them is spectrographically predominant over the outside calcium masses. In the case of stars of equal mass, the inside zero-velocity point coincides with their center of gravity, and if the inner calcium mass follows this zero-velocity point, it will have the same constant velocity as the center of gravity of the system. When the two stars are unequal in mass, as assumed in *9 Camelopardalis*, the difference in conditions of vaporization on the sides of the primary toward and away from the secondary is not so great and the development of an outside as well as of an inside calcium mass, both of which are in evidence spectrographically, is possible. Also, with increasing disparity of mass of the two stars, the inside zero-velocity point recedes toward the star of less mass. If the calcium cloud follows this point, it acquires a variable velocity, which agrees in phase with that of the secondary star.

DETERMINATION OF THE MASSES

Assuming that the spectroscopically effective calcium clouds lie about the zero-velocity points on the common radius vector of the two stars, the masses of the two stars may be determined. The mean distance of the center of the primary from the center of gravity of the system is 398,000 km. From Fig. 2*a*, it is seen that the ratio of the velocities of the primary and the inside calcium body around the center of gravity of the system is $\frac{40}{83}$. Hence this is the ratio of their distances from this point. Let $1-\mu$ and μ be

FIG. 6.—Relative positions of stars and inner calcium mass

the relative masses of the two stars M and m. In Fig. 6, let M, G, Z, and m represent respectively the primary, center of gravity of the system, inside zero-velocity point, and the secondary star. Also let $\overline{MZ}=r_1$, and $\overline{mZ}=r_2$. Since moments about G are equal,

$$M(\overline{MG})=m(\overline{mG})$$

or

$$M\tfrac{40}{83}r_1=m(1-\tfrac{40}{83}r_1) \qquad (1)$$

where $r_1=1-r_2$. Since $\mu=\dfrac{m}{M+m}$, (1) becomes after solving

$$r_1=1-\tfrac{83}{40}\mu\,. \qquad (2)$$

With the same units and notation, the following equation has been derived:[1]

$$r_2^5-(3-\mu)r_2^4+(3-2\mu)r_2^3-\mu r_2^2+2\mu r_2-\mu=0\,. \qquad (3)$$

In order to obtain an approximate value for r_2 expand (3) as a power series in $\mu^{\frac{1}{3}}$ to three terms, and equate to (2) and the following equation results:

$$0.6934\mu^{\frac{1}{3}}-0.1603\mu^{\frac{1}{3}}+2.2950\mu-1=0\,. \qquad (4)$$

Let $z=\mu^{\frac{1}{3}}$ and substitute $\left(z-\dfrac{q}{3p}\right)$ for z to eliminate the second term (p and q being the coefficients in (4) of the first and second terms). This gives, after dividing through by p,

$$z^3+0.3005z-0.4200=0. \qquad (5)$$

[1] Moulton, *Celestial Mechanics*, p. 198, *b*.

Solving (5) by Cardan's method, s is found to equal 0.6164, and μ to equal 0.2342. The corresponding value for r_2 obtained from (2) is 0.4537. A brief calculation shows that the rigorous equation (3) is closely satisfied with $\mu=0.2596$, and $r_2=0.3936$. Hence

$$\frac{M}{m}=2.85.$$

The ratio of the masses and the focal distance of the primary being known, the distance between the centers of the two stars is found to be 1,538,000 km.

Making use of Schlesinger's convenient formula[1]

$$(M+m)\sin^3 i=\tfrac{4}{10^{\circ}}\frac{(a_M \sin i+a_m \sin i)^3}{P^2}$$

where a_M and a_m are expressed in kilometers and P in days, and remembering that in the present case sin i is nearly equal to unity, it follows that

$$M+m=0.0023\odot.$$

Since the chief source of the incandescent calcium is the primary star, it is more probable that the effective calcium mass lies in a position which is a compromise between the place where it is generated and the zero-velocity point. This would increase the distance between the centers of the two stars and hence increase the mass of the system.

The parallax of this star has been derived by Slocum and Mitchell[2] and is $+0''.026\pm0''.005$.

THE BROAD LINES

The results so far have been derived without reference to the evidence given by the broad lines in the spectrum. Measures of these depend upon the lines denoted by asterisks in Table I, and the results of the double measures of them are given in Table VI. All the settings have been made upon the middle of the lines.

[1] *Publications of the Allegheny Observatory*, **1**, 145, 1910.

[2] Given at the August 1912 meeting of the Astronomical and Astrophysical Society of America.

TABLE VI

Plate	Velocity.	No. of Lines	Velocity.	No. of Lines	Velocity
I B 194	+ 7.6	2	+ 3.9	5	+ 6
229	− 2.5	3	+ 3.3	4	0
254	+ 2.9	3	+15.3	3	+ 9
285	+ 4.2	4	+11.3	5	+ 8
288	+16.5	2	+14.8	3	+16
296	+ 0.1	6	+ 1.2	5	+ 1
654	+ 7.1	5	+ 0.4	3	+ 4
663	+ 5.7	6	+ 6.7	3	+ 6
867	− 4.0	3	− 1.0	5	− 2
893	−10.1	4	− 3.9	5	− 7
1159	+ 5.7	6	+15.6	6	+ 9
1181	+13.7	4	+12.2	6	+13
1189	− 8.0	5	+ 5.5	5	− 1
1200	+28.3	5	+26.4	5	+27
1210	+24.6	6	+26.6	6	+26
1234	−10.4	6	−11.5	6	−11
1241	−10.5	7	− 4.0	5	− 7
1245	+ 5.4	6	− 5.6	7	0
1256	+20.2	6	+16.2	6	+18
1261	+15.2	5	+20.3	6	+18
1812	− 3.0	6	− 2.7	5	− 3
1821	− 6.0	5	− 3.8	6	− 5
1850	+ 2.3	6	− 3.8	3	− 1
2113	+ 0.4	7	− 4.9	2	− 2
2134	− 8.7	5	− 8.5	5	− 9
2170	− 7.4	8	−20.7	3	−14
2203	− 0.2	10	+ 7.4	10	+ 4
2278	+ 4.7	6	0.0	4	+ 2
2511	+ 6.5	7	+ 9.4	3	+ 8
2596	+ 9.9	8	+ 6.2	4	+ 8
2611	+ 2.9	8	−20.7	4	− 9
2671	+10.4	9	+ 9.3	5	+10
2808	+39.8	5	+47.9	6	+44
2814	+11.1	5	+16.7	6	+14
2831	+13.5	5	+ 9.8	3	+12
2836	+29.5	6	+34.6	3	+32
2839	+ 8.8	5	+ 6.6	4	+ 8
2843	+35.9	5	+38.2	4	+37
2846	+ 6.8	5	+ 1.8	2	+ 4
2850	− 2.3	7	− 3.4	2	− 3
2857	+11.9	9	+ 5.7	6	+ 9
2867	+19.8	8	+16.8	7	+18
2871	+ 7.5	7	+ 6.5	8	+ 7
2875	+ 9.7	7	+10.3	9	+10
2880	+16.4	5	+16.2	6	+16
2884	+17.2	8	+13.1	8	+15
2896	+10.6	7	+11.6	7	+11
2902	+ 8.3	6	+10.8	5	+10
2905	+10.2	9	+ 7.0	4	+ 9
2906	+ 8.5	7	+ 2.8	4	+ 6
2918	+ 8.0	7	+ 8.0	4	+ 8
2919	+ 7.0	7	+ 5.7	3	+ 6
2929	+15.0	5	+ 6.7	2	+12
2930	+12.0	4	+17.7	2	+15

TABLE VI—*Continued*

Plate	Velocity$_1$	No. of Lines	Velocity$_2$	No. of Lines	Velocity
I B 2932	+ 6.2	3	+ 2.5	2	+ 4
2933	+ 7.8	3	− 1.5	2	+ 3
2934	+15.9	4	+ 8.2	2	+14
2938	+ 7.7	7	+ 7.0	5	+ 7
2939	+13.3	5	+ 8.5	2	+11
2940	+ 8.3	6	+ 3.8	4	+ 6
2941	+ 7.0	7	− 4.0	2	+ 2
2942	− 7.3	4	− 7.7	1	− 7
2946	+12.0	8	+ 9.0	4	+10
2947	+ 5.1	5	+ 4.9	4	+ 5
2949	+13.7	5	+ 8.6	3	+11
2953	+ 0.2	5	+ 1.3	4	+ 1
2954	− 5.5	8	− 6.6	4	− 6
2957	+16.8	4	+20.6	4	+18
2958	−23.9	6	−31.1	2	−26
2959	− 2.2	5	− 5.2	4	− 4
2960	− 8.3	4	− 9.4	4	− 9
2961	− 3.9	6	− 2.7	4	− 3
2962	− 1.7	7	− 2.1	3	− 2
2965	+11.7	5	+16.4	6	+14
2966	+18.9	5	+18.2	5	+19
2967	+13.3	5	+20.0	5	+17
2969	+ 1.0	5	− 2.8	5	− 1
2981	− 5.5	4	0.0	4	− 3
2983	0.0	3	+ 0.1	5	0

Early in the process of measurement, attempts were made to find in the diffuse unsymmetrical lines consistent evidence of the second component. Later, after most of the plates had been obtained, they were examined again. The results were: (*a*) There are no plates showing double lines; (*b*) the lines even of the same element change radically in character from plate to plate, chiefly noticeable as unsymmetrical distortions of the order of 10 to 20 times the size of grain defects on the plate; (*c*) unless measures on the centers of the lines will give their period of oscillation, it cannot be found. Table VII shows the individual line measures and remeasures for six plates.

No period has so far been found from them, either independently of the calcium period or in attempting to find some period which is commensurable with that of the calcium lines. The conclusion of the matter seems to be: the broad lines are due to two unequal components whose velocities are of low order of magnitude. The sources of these lines exist in a disturbed state which produces

TABLE VII

λ	229		296		2203		2808		2919		2938	
	First Measure	Second Measure	First Measure	Second Measure	First Measure	Second Measure	First Measure	Second Measure	First Measure	Second Measure	First Measure	Second Measure
4026	.....	.....	+16½	.....	+ 1¹	+15¹	.....	.....	+23½	.....	+32½	.....
4089	+27°	+24½	+32½	+32¹	− 6¹	− 9¹	+25¹	+27²	+12¹	+15²	+20¹	+16¹
4097	+ 8¹	0¹	+27½	+30¹	0¹	−19½	− 7½	−10¹	+53°	+44°	+ 4½	+10¹
4101	.....	.....	.....	.....	− 5½	.....	.....	.....	+30½	.....	.....	.....
4116	.....	.....	+37½	+33¹	− 4¹	+ 7¹	+18½	.....	+18¹	.....	+24¹	+24½
4143	.....	.....	.....	.....	− 3½	− 5½	.....	+22½	− 2½	.....	.....	.....
4340	+ 3¹	+ 3¹	+18¹	+18¹	−14²	+10¹	+36¹	+18¹	+10¹	+14¹	+10²	+25½
4388	.....	.....	.....	.....	− 1½	+ 5¹	.....	.....	.....	.....	.....	.....
4471	− 1¹	+13²	+ 3½	+12²	− 2³	+ 6²	+40²	+30³	+ 5²	+ 4²	+20²	+16²
4481	.....	.....	.....	.....	.....	+ 4¹	+12¹	.....	.....	.....	.....	.....
4861	.....	.....	.....	.....	− 6²	−12¹	+14¹	+16½	−22°	.....	+17¹	.....
Weighted mean	+ 3.3	+ 9.1	+21.7	+22.8	− 4.8	+ 2.8	+27.3	+19.2	+11.7	+10.4	+17.1	+16.4
Reduction to sun	−5.81		−21.6		+4.61		+20.6		−4.7		−9.4	
Velocity	− 2.5	+ 3.3	+ 0.1	+ 1.2	− 0.2	+ 7.4	+47.9	+39.8	+ 7.0	+ 5.7	+ 7.7	+ 7.0

The small figures are line weights.

spectrum distortions that mask the possible double structure of the lines. Table VI shows a preponderance of positive velocities over those carrying the minus sign, which probably means that the broad line curve, whatever it is, lies more above the zero-velocity line than below it. This introduces a difference in the position of the γ axis as derived from the calcium lines and as derived from the broad lines. The very low pressure in the calcium masses and the relatively great pressure in the lower strata of the stellar atmosphere may give a positive and a negative correction respectively, which is sufficiently large to account for the discrepancy.

The results of this investigation may be summed up as follows:

1. A period and a velocity-curve have been derived from the H and K lines in the spectrum of *9 Camelopardalis*.

2. A physical situation has been assumed which explains the observed velocity-curve and which also takes account of the sharpness of the calcium lines as contrasted with the width and diffuseness of the other lines in the spectrum.

3. With one assumption the ratio of the masses and the sum of the masses has been computed to be 2.85 and $0.0023\odot$ respectively.

4. The proposed system has been shown to be sufficiently elastic to explain in part at least other calcium anomalies that have been observed.

5. No positive results have been obtained from the broad lines.

The general aspects of the scheme proposed in this paper for stars of this class have developed in informal discussions with Professor Frost, to whose encouragement its presentation at this time is largely due. I wish to express my appreciation to Mr. Barrett for assistance in getting the spectrograms, many of them obtained on very cold winter nights.

YERKES OBSERVATORY
JULY 1912

A HIGH-LEVEL MEASUREMENT OF SOLAR RADIATION

By FRANK W. VERY

In the *Comptes rendus* of the Institute of France for 1897 and 1898, M. Violle described an actinometer for *ballons sondes*, recommended by him to the Congress at Rome in 1879. "It is a ball of [thin] copper, blackened exteriorly and containing interiorly a thermometric apparatus whose indications can be recorded at a distance on a registering cylinder."[1] Tested in the ascension of June 4, 1898, at a height of 13,700 m, with the barometer at 118 mm and the air temperature 65° Centigrade below zero, this instrument recorded in sunshine −12°C., or a temperature-excess $\theta = 53°$ higher than the air temperature. M. Violle has been good enough to give me further details of the apparatus and method, for which I gratefully acknowledge my indebtedness. It is sufficient to say that the sun's rays are alternately excluded and caused to shine upon the instrument by the presence or absence of a double aluminum shield operated by clockwork.

The excess of temperature registered on this occasion is about three times as great as that of the blackened copper shell of a somewhat similar arrangement, the Violle conjugate bulbs, a rather sluggish instrument which, however, agrees very well with a black-bulb sun-thermometer (after applying the proper convection factor for the difference in size) when the air is perfectly calm and the conditions are steady.[2] The Violle blackened copper sphere at sea-level seldom records an excess of more than 17°

Centigrade Temperatures	Noon	12:30 P.M.	2:00 P.M.	1:00 P.M.	2:30 P.M.	3:00 P.M.
Reduced ordinary black-bulb − Air, θ_1 =	16°.9	18°.5	20°.4 (max.)	18°.8	16°.6	18°.3
Violle blackened shell − Air, θ_2 =	15.2	15.2	17.3	18.5 (max.)	17.4	18.3
Violle—black-bulb $\theta_2 - \theta_1$ =	−1.7	−3.3	−3.1	−0.3	+0.8	±0.0

[1] *Comptes rend.*, **126**, 1748, 1898.

[2] See Winslow Upton, *Report of Observations Made on the Expedition to Caroline Island to Observe the Total Solar Eclipse of May 6, 1883*, observation of the afternoon of May 2, 1883, "when the air was almost perfectly still."

or 18° C. above air temperature, representing a solar radiation, equivalent to about 1.2 cal./sq. cm. min. If the excesses were directly proportional to the radiation, the sounding-balloon reading would mean an observed thermal equivalent of 3.6 cal./sq. cm. min., but the convection losses being smaller in the rarefied air, this quantity must be reduced. It was with the design of developing a method which would be applicable to observations of static temperatures, that I undertook a thorough investigation of the theory of the Violle actinometer, presented to the Astronomical and Astrophysical Society of America at its meeting in December 1911. The conclusion then reached was that it is possible to use either the static or the dynamic method with this instrument, and with equal accuracy.

The problem of the reduction of M. Violle's high-level observation, however, is not so simple as that of the actinometer; because, instead of having a water-jacket whose temperature can be measured accurately, we are obliged to be contented with an estimate of the temperature of the environment. A similar difficulty attends the use of the Violle conjugate bulbs, where it is customary to take the difference between temperatures of bulb and air as the temperature of excess. This excess is 4° or 5° larger than would be given by the same thermometer in a water-jacket, exposed to the sun's rays through an aperture not much wider than is sufficient to include the solar angle; and this follows partly because the freely exposed instrument receives solar radiation diffused from the sky and less perfectly reflected from the earth's surface, and also partly because the temperature of the sunlit surface, even if grass-covered, is several degrees above that of the air. The heat received by the thermometric apparatus, directly from the solar radiation and indirectly from the sun's rays reflected by the surroundings, is exactly equal, when static equilibrium is attained, to the heat abstracted by contact with the air plus that removed as equivalent radiation through the air to space. The diffusion of sunlight by the sky is somewhat smaller at the height of 14 km and the radiation to space more free than at sea-level. From either of these causes the registered heat at the upper level is too small when comparison is made with a read-

ing at sea-level. On the other hand, the temperature-excess in sunshine is slightly larger when the basal temperature from which the excess is reckoned is lower.

The variations produced by these conflicting secondary influences may cause an uncertainty of a few degrees in the observed excess, but accepting the measured temperatures as the best that can be obtained under conditions which are not quite so advantageous as we could wish, though better than some to which we must submit at sea-level, we may compute the corresponding radiation by Stefan's law with Kurlbaum's constant. The blackened copper both radiates and absorbs a little less than an ideal black body and these two errors partially compensate. Whatever error remains from this cause affects equally the reading of a sun-thermometer under surface conditions. We have, then, radiation in calories per square centimeter per second at the height of 13,700 m

$$R = 1.267 \times (10)^{-12} \{(261)^4 - (208)^4\}$$
$$= 0.005878 - 0.002372 = 0.003506.$$

It is not necessary in the static method to know the water-equivalent of the radiating mass, because the condition of the radiating surfaces and of the surrounding medium alone affect the comparison; but among the conditions as to surface we do need the dimensions of the apparatus, since the radius of curvature of the sphere (in this case 2.5 cm) enters into the computation of the convection. Inserting this value in a formula which represents the result of comparisons of thermometer readings in inclosures with air at ordinary pressure and exhausted,[1] I obtain the following provisional value of the combined losses per sq. cm. of surface and second of time:

Convection+molecular penetration	0.00843
Radiation	0.00351
Sum	0.01194

Since the sum of the losses per second represents emission from a spherical surface, this number must be multiplied by four to give

[1] *Publications of the Astronomical and Astrophysical Society of America*, **2**, 90.

the solar radiation in the sectional beam which produces the heat of the steady interchange, and this being further multiplied by 60 to reduce to minutes, gives

2.86 gram calories per sq. cm per minute

for the observed solar radiation-equivalent.

The transference of heat through the air between surfaces separated by a narrow space occurs mainly through penetration of gaseous molecules, the mass movements of convection currents being impeded by viscosity and adhesion of the air to the neighboring solid surfaces. There is greater freedom of motion in a wider inclosure, and more of the motion of individual molecules is transferred to simultaneous group movements in convection currents. The complete formulas take into account the actual pressure and temperature, as well as the dimensions of the apparatus, and give a sum of penetration and convection which is probably not very different from that in the free air,[1] although the relative importance of the two constituents is no doubt changed. The measures on which the computation is founded were necessarily obtained in a narrowly circumscribed inclosure within which a vacuum could be produced. In a larger inclosure the convection would have been greater and the penetration smaller, the radiation remaining unchanged. It is not easy to foresee just how these changes would affect a body in free air, and I can only call the result provisional, but it certainly points to a value of the solar constant at least as great as 3, and probably very nearly 3.5 cal./sq. cm min.

M. Violle's sounding-balloon observation is a notable one, but above its altitude of nearly 14 km are many more kilometers of absorbent atmosphere. Blair and the Mount Weather observers found the temperature still increasing up to nearly 30 km where −41° C. was recorded, or 24° higher than the air temperature in M. Violle's measurement. Aqueous vapor in appreciable quantities is found at these great altitudes, showing that the chief atmospheric absorbent is still present, and thus there is no reason

[1] In calm free air, the excess of a black-bulb sun-thermometer above the temperature of surrounding surfaces does not differ much from that of the same thermometer in an actinometer inclosure over the temperature of the water-jacket, after eliminating the effect of sky radiation.

to doubt that aqueous vapor is chiefly responsible for the absorption of solar radiation by which the great temperature-inversion of the upper non-adiabatic layer is produced, just as the presence of an excess of aqueous vapor has been found to be associated with minor temperature-inversions in the lower atmosphere. On September 11, 1910, the Weather Bureau sounding-balloon ascen-

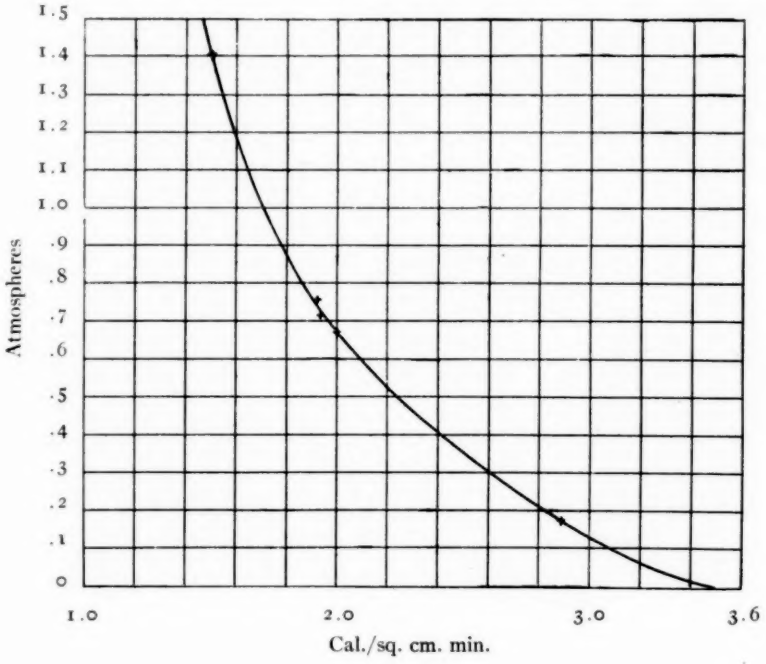

FIG. 1

sion from Huron, S.D., gave a temperature of $-25°.5$ C. and a relative humidity of 56 per cent at an altitude of 24,899 m.

These observations extend the limits of the aqueous atmosphere to much greater altitudes than some meteorologists have thought to be possible. The form of the temperature-curve indicates that the approximately isothermal layer, and therefore the aqueous atmosphere whose absorption of solar radiation is the chief factor in producing it, extends to perhaps something like 80 km. Thus it is highly probable that the increment of solar radiation, observed

up to 14 km, will continue with only a gradually diminishing rate up to the limits of the "isothermal" layer.

In conclusion, an independent estimate of the solar constant is given, taking observations of solar radiation at three very different heights in the atmosphere, namely, a thermal equivalent of 1.5 cal./sq. cm min. at sea-level for transmission under best winter conditions by an air mass of 1.4 atmospheres, 2.00 cal. corresponding to an air mass of 0.674 and an altitude of 4420 m, and 2.86 cal. in the isothermal layer at 13,700 m. The middle value is Keeler's observation on the summit of Mount Whitney, and there are also plotted for comparison Nanry's reading at Mountain Camp, Mt. Whitney, from Langley's *Solar Researches* with trifling corrections by myself, together with an entirely new reduction by me of Violle's measurement at the summit of Mont Blanc. All of these measurements were made with the Violle actinometer, an instrument of unsurpassed precision.

The sounding-balloon observation is less precise but furnishes a precious indication of the power of the solar rays at an altitude never before reached in such measures. The high-level observations were made in summer, but under conditions of local temperature and humidity which resemble those of the winter observations chosen to represent sea-level. Thus we have a fairly homogeneous series. A smooth curve passed through these three points (Fig. 1) meets the axis of X at 3.5 cal./sq. cm min., which is the solar constant of radiation.

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A CRITERION OF ACCURACY IN MEASUREMENTS OF ATMOSPHERIC TRANSMISSION OF SOLAR RADIATION

By FRANK W. VERY

It is to James Forbes that we owe the first recognition that the absorption of solar radiation by the earth's atmosphere not only varies with the changing composition of the atmosphere from summer to winter, but is inconstant even during a single day of cloudless sky, and that the coefficient of absorption of the transmitted rays, from which the more absorbable rays have been sifted out, goes on diminishing more and more as the atmospheric mass already traversed becomes greater. At least this is what ordinarily takes place. We shall see that there are exceedingly rare exceptions to this law which are of the utmost importance.

By a further extension of his principle, Forbes arrived at the conclusion that the intensity of the transmitted rays tends to reach a definite value after passing through an infinite mass of atmospheric air. This variability of the coefficient of absorption by a unit layer of air was attributed to the non-homogeneity of the solar radiations which are unequally absorbed, so that the sifted sunbeam becomes progressively richer in more transmissible radiations the greater the atmospheric mass which they have traversed. The importance of the principle in question was recognized by Radau in his admirable little treatise, *Actinométrie*, and has been discussed at length in the writings of Crova and Langley.

In addition to this variation of radiant absorption by a unit layer with changing air mass, attributable to the non-homogeneity of the solar radiation, there is a further variation due to the systematic change in the absorbent quality of the air by which a unit layer, receiving identical radiation, transmits this radiation less freely in the middle of the day. The cause of this variation of absorbent quality has been discussed in my paper "On the Need of Adjustment of the Data of Terrestrial Meteorology and of

Solar Radiation, and on the Best Value of the Solar Constant."[1] In the present communication I propose to consider the means by which we may recognize this variation of atmospheric quality and in some cases apply corrections for the same.

If it were possible to measure strictly homogeneous radiations, and if occasions could be found when the atmospheric quality remains unchanged between high-sun and low-sun observations, we could apply Bouguer's equation

$$R = Ap^{\epsilon},$$

in which R = solar radiation received normally on the unit of area at the surface of the earth, A = the solar constant, p = the coefficient of transmission, assumed to remain unchanged during the day, and ϵ = the air mass taken as unity for sea-level and normal pressure, so that, if B is the observed barometer reading at the place of observation and ζ the sun's zenith distance, $\epsilon = (B/760)$ sec ζ, until zenith distances greater than 56° are reached, beyond which a small correction is required.

In the *Annales de chimie et de physique* for 1888 (Ser. 6, t. 14, p. 541–574), M. Crova described the results obtained with his actinograph thus:

> The constant character of the numerous curves which we have studied consists in a permanent fluctuation of the intensity of solar radiation [as received at the surface of the earth]. All bear witness to rapid oscillations of intensity, and it can be affirmed without exaggeration that the needle of the actinometer is continually in motion. These oscillations are translated on the curves into sinuosities from which none are exempt, but which in calm, warm, and moist weather are so close together that they become confounded, giving a smooched appearance to certain parts of the curve through a considerable breadth.[2]
>
> In summer, at Montpellier, the part of the curve which corresponds to the rising of the sun is, in general, clear and regular; but soon the oscillations begin and attain, toward 9^h in the forenoon, such an amplitude that the curve becomes blurred in consequence of the rapidity of the oscillations, then it is strongly depressed, and in the afternoon the radiation, for an equal altitude of the sun, is smaller than in the morning.[3]
>
> In winter, above all after severe frosts, the curve is relatively regular and approaches the theoretic symmetry. It is especially in great cold and with

[1] *Astrophysical Journal*, **34**, 371, 1911.

[2] *Op. cit.*, p. 543.

[3] *Ibid.*, p. 545.

high winds that the curves present the greatest regularity and the feeblest oscillations. [Thus the fluctuations must be attributed to the interception of solar radiation by ascending columns of moist air which are produced by convection and evaporation from a moist and heated soil. Strong wind mingles the contents of the convective striae indiscriminately, and cold diminishes their vapor-content.] Most of the curves show a depression which is frequently quite marked toward noon [or at the time of day when convection is most powerful]. The radiation then increases again to diminish afterward until sunset.[1]

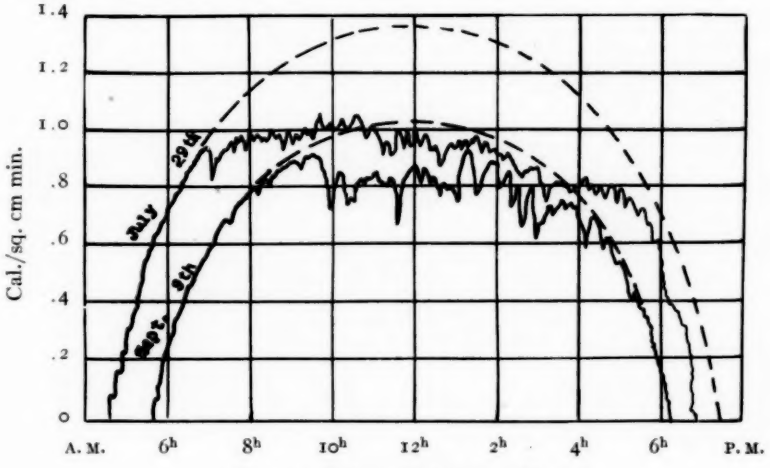

FIG. 1.—Actinograph curves (Crova)

On only 1.9 per cent of the days in 1886 were approximately symmetrical diurnal curves of insolation obtained, and still fewer in 1885 and 1887. Thus convection increases the depth through which an especially absorbent ingredient of the atmosphere is dispersed, and to such an extent that the effective absorbent mass is notably greater than the theoretical ϵ in the middle of the day, at the same time that the penetration of the rays suffers an exceptional diminution; that is to say, p is exceptionally small and ϵ larger than is assumed, or, since p is a fraction and ϵ greater than unity, the quantity p^{ϵ} is diminished to a still greater extent.

Fig. 1 shows two curves which were obtained by Crova with his registering actinometer on July 29 and September 9, 1886. The July curve gives an indication of the large midday loss of

[1] *Op. cit.*, p. 545.

insolation due to the temporary increase of convection of moisture-bearing air-columns. This loss is to be distinguished from the equably distributed loss which still affects the restored curve of insolation, which is represented in the figure by the dotted line and which corresponds to a radiation equivalent of 1.4 calories at the maximum. The distortion of this curve begins as early as 7^h A.M. The curve of September 9 is not affected until after 9^h A.M., and its average departure from the restored curve is much less, although the average transmissibility of the air was not as large as it frequently is.

In addition to the variations of atmospheric transmission through changes in the distribution of its moisture by convection, there is a further change in the molecular constitution of some of the atmospheric ingredients by which especial obstruction to the passage of solar radiation is induced in the middle of the day when the sun is highest. The effect is very much greater in summer than in winter, and there is reason to believe that it is caused by an ionization of the upper air by the sun's ultra-violet rays by which chemical changes ensue, particularly the production of hydrols; but a portion of the diurnal change comes simply from the more extensive dispersion of aqueous vapor and dust into the upper air during the middle of the day. The great fact of this midday variation of atmospheric quality is so obtrusive that it cannot be ignored, and the Bouguer formula must be modified in order to cover the actual facts. This M. Crova has already done in a series of admirable analyses which have not received the recognition they deserve.

M. Crova's method of reduction is not a rational one, but results from the application of a mathematician's devices to the study of the form of the curve of observation. He says:

> I can then consider the curve of calories, given in function of the thicknesses, as composed of an infinity of logarithmic arcs in which A is constant, while p varies in a continuous manner from one to the other, increasing with ϵ and serving as the envelope of the arcs. Under this hypothesis, it will be easy to measure the coefficient of transmission corresponding to a determined value of ϵ, by constructing a tangent at the point whose abscissa is ϵ and measuring the value of the subtangent.[1]

[1] *Ann. de chim. et de phys.*, Ser. 5, **11**, 496, 1877.

The coefficient of transmission thus calculated represents the ratio of intensity of the radiation transmitted through a thickness equal to unity to the intensity of the radiation before this transmission, admitting that this absorption changes only the quantity and not the composition of the incident beam.[1]

This is of course not true except for absolutely homogeneous rays which are never measured even by the spectrobolometer, since the structure of a Fraunhofer line is itself most complex; but with certain restrictions Crova's assumption may be approximately true even for the still more complex total radiation, provided we consider an absorbent layer of limited thickness within which the quality of the air is sufficiently constant, and from which p may be derived for a unit layer of like composition. The permissible depth of this assumed layer is itself variable, if a reasonable approach to accuracy is to be maintained, and further corrections are necessary before the process can be applied with any certainty to the circumstances of a complex atmosphere.

If p were really constant throughout the radiation-curve, we should have transmission $= R/A = p^{\epsilon}$, or the transmission by unit layer anywhere in the atmosphere should be $T = p$, instead of which we observe variable values of T according to the previous history of the radiation, or the level which it has reached in the atmosphere, such as $T = p_1$, $T = p_2$, or in general $T = p_x$.

With both p and ϵ variable, the variation of the solar radiation in a thin layer of air ($\Delta\epsilon$) is

$$R_1 - R_2 = A p^{\epsilon} \log p \Delta\epsilon + p^{\epsilon-1} \epsilon \Delta p .$$

This is not a practicable formula and Crova has therefore sought some simple approximation which will be applicable. The original parabolic or hyperbolic function is thrown into a form

$$R = A/(1 + \frac{m}{c}\epsilon)^{1/m} ,$$

where m and c are numbers to be found by observation. Bouguer's equation may be derived from this as a special case.

Since m and c are very nearly identical, the ratio m/c being sometimes greater and sometimes less than unity, and since these

[1] *Op. cit.*, p. 497.

numbers have no physical significance but are merely of use in determining the subtangents of the radiation-curve, the last equation may be written

$$R = A/(1+\epsilon)^{1/m}.$$

Differentiating, we have

$$\frac{dR}{R} = -\frac{1}{m} \cdot \frac{d\epsilon}{1+\epsilon},$$

and

$$\left(\frac{dR}{d\epsilon}\right)/R = -\frac{1}{m(1+\epsilon)};$$

but the first member is the reciprocal of the subtangent of the radiation-curve, whose sign must here be taken negative because R decreases as ϵ increases; and the subtangent is also the reciprocal of the transmission T corresponding to air mass ϵ. Hence the transmissions of the sifted rays at the stage represented by ϵ is given by the equation

$$\log_{(10)} T = -\frac{M}{m(1+\epsilon)}$$

(true only for rays of a special quality which occur at this point in the path), or

$$T = e^{-\frac{1}{m(1+\epsilon)}}.$$

Hence T is a variable which can be studied from the subtangents of the radiation-curve, which can be read graphically from the $R\epsilon$ curve, giving a series of straight lines whose equations are

$$S = (Rd\epsilon)/dR = c + m\epsilon,$$

which expresses the fact that, through a considerable range of the curve, the subtangents increase by a nearly constant quantity (c) for successive increments of ϵ by unity.

Values of m may be obtained by the equation

$$m = \log\left(\frac{1+\epsilon_1}{1+\epsilon_2}\right) \div \log\left(\frac{R_2}{R_1}\right),$$

which commonly gives a series of varying numbers, so that we are no nearer arriving at a single fixed value of the solar constant

than before, unless some way of harmonizing these conflicting data can be found.

Two methods are possible: (1) If on any one day the atmospheric quality is sufficiently constant to give equable values of m throughout the day, we may at once derive the solar constant by the equation

$$A = R(1+\epsilon)^{1/m}.$$

This corresponds to an ideal equation

$$A_1 = R^m(c+m\epsilon),$$

in which the differences of the subtangents for successive differences of one unit in ϵ are all equal to the constant c. In this equation, A_1 is not the solar constant, but the latter can be obtained from it by the equation

$$A = \left(\frac{A_1}{c}\right)^{1/m}.$$

(2) Failing to obtain an ideal series, we must look for some criterion for estimating the value of the observations from which m is derived, and either reject all values which do not conform to the criterion, or apply suitable corrections, if such can be found.

The variable transmission (T) is represented by Crova by a logarithmic curve

which has for its asymptote a straight line parallel to the axis of X at a distance equal to unity, and whose ordinate at the origin is $e^{\frac{1}{c}}$. This coefficient varies therefore from a minimum value $e^{\frac{1}{c}}$, which represents the transmission of the rays which arrive at the limits of our atmosphere, and a maximum equal to unity, which corresponds to the case where, the rays having traversed an infinitely great thickness of air, their intensity approaches zero, and the transmission is made without loss.[1]

This does not exactly express the case. The transmission must begin with unity, fall rapidly to a minimum, and then increase slowly, eventually approaching unity once more. The transmission computed by Crova's method is still only an apparent one, because it rests on the assumption that "this absorption changes only the quantity and not the composition of the incident beam."

[1] *Op. cit.*, p. 498.

Thus the value deduced for the radiation at the limit is not the solar constant, but the quantity $A-B$, where B is *lost* radiation. There is, of course, an attempt to make allowance for changing quality, but the variation of the resulting constant from 1.865 to 2.703 (see Crova's paper in the *Annales*, Ser. 6, t. 14) shows as wide a range as is given by a simple graphical estimate on rational principles. There is little advantage in the rigorous mathematical treatment of such data, unless the operations can clarify our reasoning.

Using the more exact equation for the transmission of the sifted rays which have already passed through a thickness of ϵ atmospheres at the stage of absorption corresponding to ϵ, we have

$$T=e^{-\frac{1}{\epsilon+m\epsilon}}$$

which varies from $e^{-\frac{1}{\epsilon}}$ for $\epsilon=0$, to unity for $\epsilon=\infty$. We wish to follow the variations of the quantities $\frac{1}{m}$[1] and T.

An example of an observation on a single day with nearly constant values of $1/m$ has been given us by M. Savélief, who secured the measures on December 28, 1890, at Kief in Russia. The meteorological conditions are thus described:

> According to the *Bulletin de l'observatoire physique central de Saint-Pétersbourg*, we have had before the 26th of December, in European Russia, several days of severe cold with the wind from the east, and at Kief a north-east wind under the influence of a barometric maximum over central Russia. The ground was covered with a thick layer of snow, and the atmosphere at Kief must have contained a very small quantity of water and of dust.[2]

The temperature varied between $-21°.9$ C. at 7^h A.M., $-18°.6$ at 1^h P.M., and $-19°.9$ at 9^h P.M. Barometer $=766$ mm. Pressure of aqueous vapor $=0.8$ mm. Wind from 2 m./sec. to calm. Sky "of a pure blue without any visible cloud from morning until evening."

From the mean value of $1/m=0.647$, m is 1.55 which multiplied

[1] M. Crova sometimes uses this symbol and sometimes places $p=1/m$, thus using p in two different senses. To avoid confusion, I have kept to a single significance for each symbol.

[2] *Ann. de chim. et de phys.*, Ser. 6, **25**, 570, 1892.

into ϵ, and with the observed subtangents (S), gives the remaining figures in the following table. Column (4) contains the values of $c=S-m\epsilon$; (5) the transmission of the sifted rays, $T=e^{-\frac{1}{c+m\epsilon}}$; (6) the penetration of the total solar radiation, or p, per unit mass, computed from the equation $p=\sqrt[4]{R/A}$, taking $A=3.5$; (7) the remainders $=p-T$; (8) the final penetration of the rays, p^{ϵ}.

(1) ϵ	(2) $m\epsilon$	(3) S (obs.)	(4) c	(5) T	(6) p	(7) $p-T$	(8) p^{ϵ}
0	0	(2.29)	2.29	0.647	1.000	+0.353	1.000
0.5	0.775	2.87	2.10	.706	0.575	− .131	0.759
1	1.55	3.48	1.93	.750	.637	− .113	.637
2	3.10	4.73	1.63	.810	.699	− .111	.489
3	4.65	6.08	1.43	.848	.740	− .108	.406
4	6.20	7.54	1.34	.876	.770	− .106	.351
5	7.75	9.13	1.38	.896	.792	− .104	.311

The curve from which the subtangents of the above table have been read is obtained by Crova's method, that is to say, the extension to the axis of Y must conform to the requirement that the equation of T is to be logarithmic. The agreement with theory within the range of the observations, which cover six units of air mass, is very nearly perfect. T increases with ϵ continuously, but p diminishes at first and then increases. The apparent transmission of the sifted rays obtained by the theory can only agree with p at a single air mass between $\epsilon=0$ and $\epsilon=0.5$. It is of course impossible that the transmission should be $T=0.647$ for $\epsilon=0$, as the equation for T literally requires, and the only rational interpretation of the result is that the value of T suddenly drops from unity at $\epsilon=0$, to a minimum which is equal to $1/m$ at some very small air mass, beyond which the curve of T is in substantial agreement with the adopted logarithmic equation. This interpretation is represented in the curve of T which I have drawn in Fig. 2. This mode of viewing the matter could also be represented by changing the form of the equation, but it would no longer be suitable for computation.

The curve of R for Savélief's observations gives $R=2.23$ cal./sq. cm min. for $\epsilon=1$. This large value need not surprise us, for if it were possible to have the moisture removed from the atmos-

phere above a sea-level tropical station to the extent represented by a temperature of 20° Centigrade below zero, and with the concomitant absence of moisture implied by a pure cloudless sky with this surface temperature to have also an air free from dust, such as existed over the Russian snow fields, the vertical midday sun would shine with tremendous power.

Under less favorable conditions, Crova's method usually gives

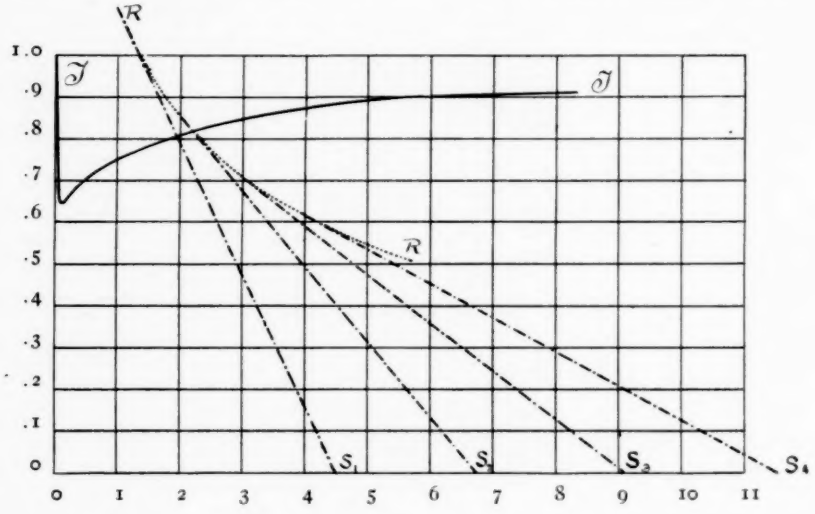

FIG. 2

larger values of $1/m$ than 0.647 and smaller values of A than 3.5, but as M. Savélief says: "It may be admitted that in this case a portion of those radiations most absorbable by the atmosphere has been entirely arrested, and consequently those which remain correspond to a higher coefficient of transmission."[1] This explanation of the small apparent values of the solar constant (A) which are frequently obtained with unsuitable atmospheric conditions, but which really relate to the quantity $(A-B)$,[2] also accords with the wide variation in the results found by different observers, using reliable instruments, but hampered by divergent atmospheric states.

[1] *Op. cit.*, p. 574.

[2] See my paper "The Solar Constant" (*U. S. Weather Bureau Publication No. 254*, pp. 26–28, 1901), where this subject is illustrated.

M. Savélief's final conclusion, just quoted, appears to me to be correct. The radiation which has vanished may belong to very different regions of the spectrum. Thus a large amount of aqueous vapor with little dust may cut off infra-red rays to an exceptional extent, while a dry and dusty air may unduly deplete the radiation of short wave-length, and the distribution of the original radiation in the different regions being quite unlike, the resultant effects will be proportionally different. Variations of other atmospheric ingredients will affect their own special regions of the spectrum. Hence p may vary widely with little corresponding change in the total intensity of the transmitted radiation, or an apparent p, deduced by incorrect methods, may remain steady, while the computed A, which is really $A-B$, varies considerably.

When the cause of error due to reflection from bright clouds has been eliminated, it is correct to select, as M. Savélief does, the highest records of the actinograph, rather than mean values, for computation. The figures which he gives for the month of July, 1.26 and 1.46 cal. at 6^h and 7^h A.M., are extraordinarily large for the season and hour, and they appear to have been the only summer readings which could match his remarkable winter one, while no other instrument than an instantaneous one could catch these exceptional moments of maximum atmospheric transmission. We are, indeed, left a little in doubt whether such casual momentaneous results are genuine, or whether they may possibly be instrumental errors; but this doubt does not affect the consistent measures of an entire perfect day.

The opening words of this author: "It is principally during the winter that the recording actinometer gives the most interesting results in the way of a determination of the solar constant and of atmospheric transmission," are fully justified. The square-shouldered form of the diurnal curve of solar radiation, which is observed on high mountains, requires small values of $1/m$. The usual form of the curve at sea-level agrees better with values of $1/m$ greater than 0.6, if the missing midday portion of the curve is supplied as shown in the broken lines of Fig. 1. Such restorations are assuredly necessary, but to attempt them as a preliminary to reducing the observations for the solar constant would be a very

problematical procedure. I have therefore sought another mode and have compared the values of $1/m$ in Savélief's series of December 28, 1890, with the mean of two unusually extensive series by Mr. H. H. Kimball on November 29 and December 1, 1909, which were continued until the air mass was over 26 atmospheres.[1] The values of $1/m$, computed from these measures, are given in the next table, after which follow the values of $1/m$ from Kimball's mean results for five years, including measurements made in every month. These are larger than the selected results.

VALUES OF $1/m$

	Atmospheres									
	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16
November 29	0.828	0.830	0.930	1.14	0.899	0.980	0.988	1.03	1.15	1.17
December 1	.678	.779	.863	0.971	.989	1.23	1.25	1.32	1.46	1.56
Mean	0.753	0.804	0.897	1.056	0.944	1.105	1.119	1.175	1.305	1.365
	16–17	17–18	18–19	19–20	20–21	21–22	22–23	23–24	24–25	25–26
November 29	1.45	1.28	1.39	1.41	1.58	1.68	1.78	1.75	1.81	1.87
December 1	1.57	1.69	1.80	1.89	1.97	2.02	2.03	2.16	2.27	2.39
Mean	1.510	1.485	1.595	1.650	1.775	1.850	1.905	1.955	2.040	2.130

1905–1910

Atmospheres	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11
Mean $1/m$	0.515	0.574	0.633	0.692	0.751	0.810	0.869	0.928	0.987	1.046

The value of $1/m$ for 9–10 atmospheres in the first part of this table is excessively large, and I have substituted for it the mean of the three pairs on either side, or $1/3\ (0.921+0.955+0.936)=0.937$.

Comparing results from the measures of Savélief and Kimball between 4 and 10 atmospheres for a mean of 7 atmospheres, I have smoothed the Kimball values a little at 4–5 and 5–6 atmospheres, taking the mean difference of $1/m$ for one atmosphere

$$(6\text{–}16\ \text{atm.}),\ (1.365-0.753)/10=0.061,$$
$$(16\text{–}26\ \text{atm.}),\ (2.130-1.510)/10=0.062.$$

[1] *Bulletin of the Mount Weather Observatory*, **3**, Part 2, pp. 90–91, 1910.

This gives a smoother curve with practically identical mean values.

Atmospheres	Savélief	Kimball
9–10	0.669	0.937
8– 9	.693	.897
7– 8	.657	.804
6– 7	.612	.753
5– 6	.605	.692
4– 5	.646	.631
Mean for 7 atmospheres	$1/m=0.647$	$1/m=0.786$

Applying the mean values of $1/m$ to the radiations for the middle of the series ($\epsilon=7$) in these limited sections of the entire curve, I obtain by the equation.

$$A=R(1+\epsilon)^{1/m};$$

from observations by Savélief,

$$A=(8)^{0.647}\times 0.94=3.606;$$

from observations by Kimball,

$$A=(8)^{0.786}\times 0.72=3.698$$

in calories per sq. cm per minute.

The form of the equation for A requires that the exponent of $1+\epsilon$ shall be a fraction, and the ideal day is one in which the value of $1/m$ remains constant as in the measurement by Savélief; but the above comparison shows that Crova's method is also capable of giving consistent values for the solar constant from the results of much less auspicious days, provided *the criterion that $1/m$ must in no case exceed unity* is applied, and with the further proviso that *midday observations which are affected by the usual midday depression must be omitted.*

Kimball's own reduction of his observation of December 1, 1909, by the erroneous method which takes no account of the diurnal change of atmospheric quality, gave $A=2.017$. The observation by Savélief is the most perfect on record and bears internal evidence of the purity of the atmosphere through which

he obtained his readings. I have therefore chosen this measurement as a standard by which less favorable ones may be judged. The application of this test shows that at about 5 atmospheres Crova's exponent ($1/m$) usually becomes larger than the value (0.647) which Savélief found for it, and that under ordinary conditions $1/m$ is apt to exceed unity at about 10 atmospheres, and to approach or exceed 2 near sunset, which would give impossible values of A. If, however, we assume $A=3.5$ and calculate $1/m$,

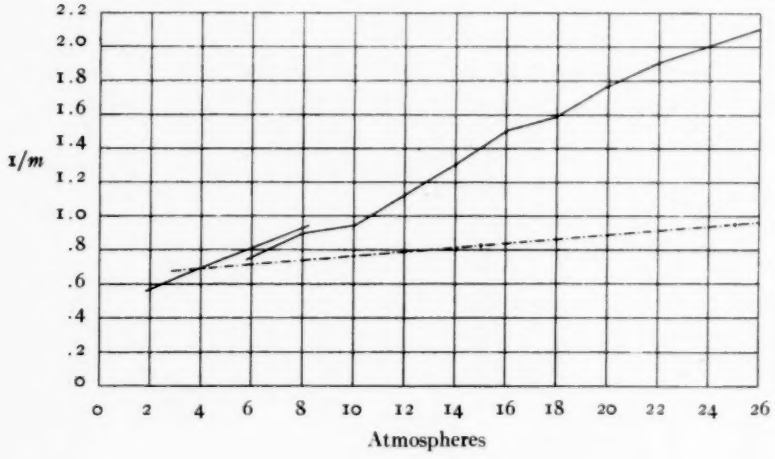

FIG. 3

for example, for $\epsilon=15$, $R=0.355$, and $\epsilon=25$, $R=0.158$, we get $1/m=0.825$ and 0.951, which are permissible values. Fig. 3 shows the relation between the apparent and the real values of this exponent.

Observations by photometry in the green of the solar spectrum, carried to a very low altitude of the sun, show that visible rays in this part of the spectrum continue to diminish indefinitely. These observations by Kimball, and some which I have made in the infrared spectrum indicate, however, that there are remnants of infrared radiation for which the air is almost perfectly transparent. When the transmitted beam approaches this condition of a readily transmissible remnant, our formulae all fail, except in the rare

condition of a very pure and dry atmosphere such as Savélief had. In this case $1/m$ probably increases very slowly; but a constant $1/m$ would give progressively diminishing values of A for increasing air mass. The logarithmic law gives a constant value of A, but an erroneous one, and no single, simple treatment will cover the whole range of varying conditions without some progressive modification on rational principles. It seems to me that the above proves conclusively that empirical mathematical formulae do not apply unless the conditions are exceptionally simple.

I might, of course, take the agreement of Kimball's December observation with that of Savélief (both being computed for the range 4–10 atmospheres by Crova's formulae) as indicating that it is possible to reach an accurate value of the solar constant by confining the computation within the range for which the formulae are applicable; but it seems a little arbitrary to fix the limits in this way. Still I do think that something can be said in favor of such a procedure, because the measurement of atmospheric action ought to be made with the sun far enough from the meridian to insure that chance fluctuations will not be larger than the thing to be measured. The records of the Crova actinograph show that chance fluctuations have their greatest magnitude in the middle of the day. The rule that only maximum points shall be considered is inapplicable to the midday portion of summer curves, since even the maxima fall far short of a true restored curve; but the portion of the diurnal curve between the limits of four and ten atmospheres conforms tolerably well to the conditions needed for a determination of its slope and general form, and, as a rule, it would seem to be the best part of the curve to select for computation.

The foregoing treatment of Kimball's observation is a rational one. It simply insists that we must not take an arbitrary formula and apply it literally and recklessly to observations which are affected by sources of error not contemplated in the scheme of reduction, but that it is preferable to follow a procedure which recognizes the existence of demonstrable sources of error, tries to detect them, and applies corrections for them.

My determination of the solar constant from the sounding-balloon measurement of M. Violle, described in my paper in the preceding article,[1] is in good agreement with Savélief's observation. By the treatment recommended here, Kimball's observations are also brought into accord, not only without doing violence to the data, but by the application of precautions which are obviously

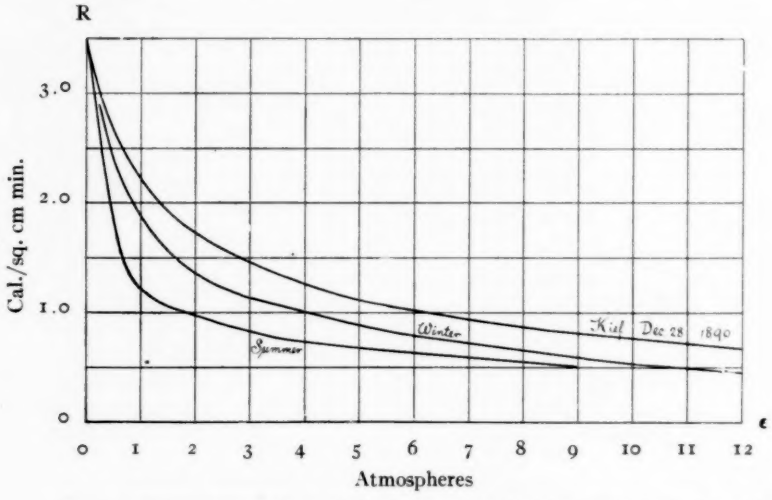

FIG. 4.—From observations of Kimball at Washington, D.C., and of Savélief at Kief, Russia.

required. If any use is to be made of solar observations secured under imperfect conditions (and the vast majority of solar actinometric observations is in this case), some such mode of selection and reduction as the above is an absolute necessity.

It will conduce to a recognition of the general facts of the atmospheric depletion of solar radiation, if we exhibit a series of radiation-curves for summer and winter, beginning with the unrivaled Kief measurement which is shown in the upper curve. The middle curve represents such a course as would be followed in less extreme winter weather, and the lowest gives a selection of measures by Kimball in June at Washington, D.C. (Fig. 4). With a general

[1] "A High-Level Measurement of Solar Radiation," *Astrophysical Journal*, **37**, 25, 1913.

parallelism from four atmospheres onward, the curves show considerable difference of form for air masses less than two atmospheres. The bend of the summer curve in this part of its course is much sharper and the ascent from one atmosphere to the radiation at the atmospheric limit is very steep at this season.

WESTWOOD ASTROPHYSICAL OBSERVATORY
WESTWOOD, MASS.
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STANDARD WAVE-LENGTHS IN THE ARC SPECTRUM OF IRON, REDUCED TO THE INTERNATIONAL UNIT

II. FROM λ 5328 TO λ 6495

By F. GOOS

In a former paper I have published the wave-lengths of the iron spectrum from λ 4282 to λ 5324.[1] The present paper contains the results of a continuation of that investigation carried out in the same manner. In order that the exposure time should not be too long, from λ 5371 on, the fifth order instead of the sixth was used. The exposure time at λ 5500 amounted to 7 minutes; at λ 5800, 12 minutes; at λ 6100, 17 minutes; and at λ 6400, 20 minutes, upon Wratten & Wainright Panchromatic-A plates.

In the yellow-red part of the spectrum in which the *Fe* lines are few and rather poor, some *Ni* lines have been added. In order to produce the *Ni* spectrum, a nickel tip was melted upon the under electrode of the ordinary iron arc, before the exposure was made, by holding a piece of pure nickel wire in the arc flame. In this way there existed, to a certain extent, both a *Fe* and a *Ni* electrode. While the photograph was being made for a third of the exposure, *Ni* was made the negative pole; then, by a reversal of the current, *Fe* was made the negative pole, during the remaining two-thirds of the time, so that the *Ni* lines should have the proper strength in comparison with the *Fe* lines. In the red part of the spectrum the ordinary iron arc was again used, but the reversal of the current was still continued, and the exposure was made half of the time with the negative pole below and the other half with the negative pole above. In this way the spectral lines maintained throughout their whole lengths a uniformly sharp and fine appearance, while otherwise they might easily appear wedge-shaped, corresponding to the greater intensity of the arc in the vicinity of the negative pole. The strength of the current, as before, amounted to about 7 amperes at 110 or 220 volts.

[1] *Zeitschrift f. wiss. Phot.*, **11**, 1, 1912; *Astrophysical Journal*, **35**, 221, 1912.

Each plate covers a range of approximately 300 Å., with about nine normals of the second order. The thirty-three plates measured cover the following regions of the spectrum. The first seven have been repeated from the earlier investigation for the sake of reference.

Plate	From λ	To λ	Plate	From λ	To λ	Plate	From λ	To λ
31	5110	5371	42	5498	5763	53	6003	6318
32	5167	5406	43	5498	5805	54	6027	6335
33	5167	5435	44	5570	5893	55	6065	6394
34	5192	5435	45	5616	5935	56	6138	6431
35	5233	5498	46	5659	5953	57	6138	6431
36	5267	5507	47	5761	6027	58	6192	6495
37	5302	5570	48	5761	6065	59	6231	6546
38	5371	5659	49	5805	6138	60	6265	6546
39	5371	5659	50	5858	6192	61	6318	6593
40	5406	5659	51	5935	6231	62	6394	6593
41	5498	5763	52	5953	6265	63	6394	6593

In addition to the normals of the second order, established by the international resolution, some other lines, especially *Ni* lines in the yellow-red part of the spectrum, must be introduced, lines which have been measured by the interference method by only one or two observers. In the table of wave-lengths at the end of this paper, these lines are indicated by an asterisk in the fifth column. The normal λ 5455.6 must be rejected because a *Fe* line only 0.18 Å. distant almost blends with it, and seriously reduces the accuracy of its measurement. The agreement of the normals is in general good. It is only in the region from λ 5761 to λ 5953, and especially in the case of the *Ni* normals, that great discrepancies occur, but these are not so great as Kayser found.[1] The following table

I.N.	Kayser	Goos	Kayser—I.N.	Goos—I.N.	Element
I.Å.			I.Å.	I.Å.	
5371.495	.490	.495	−0.005	0.000	*Fe*
5615.661	.667	.663	+ 6	+ 2	*Fe*
5658.836	.846	.835	+ 10	− 1	*Fe*
5760.843	....	.850	.....	+ 7	*Ni*
5805.211	.212	.204	+ 1	− 7	*Ni*
5857.760	.753	.761	− 7	+ 1	*Ni*
5892.882	.897	.888	+ 15	+ 6	*Ni*
5934.683	.668	.673	− 15	− 10	*Fe*
5952.739	.739	.744	0	+ 5	*Fe*
6003.039	.029	.041	− 10	+ 2	*Fe*
6430.859	.848	.860	− 11	+ 1	*Fe*

[1] *Zeitschrift f. wiss. Phot.*, **9**, 173, 1911; *Astrophysical Journal*, **32**, 217, 1910.

contains under the heading "I.N." those normals, in the case of which large differences have been found either by Kayser or by me. The greatest departure in my determinations is 0.010 Å. for the line λ 5935, whereas Kayser finds here a difference of 0.015 Å. It looks as if, in the adjustment of the normals, I had divided this difference between the two lines λ 5935 and λ 5953, for both Kayser and I find the sum of the differences of the wave-lengths of the two lines to be the same. I have not confirmed his very large discrepancies for the lines λ 5659, λ 6003, and λ 6430.

Though the results for the normals of the second order obtained by different observers show marked differences, this tendency is even more conspicuous in the new normals of the third order as shown by a comparison of my measures with those of Kayser, especially in the region from λ 5400 to λ 6200. Although these differences frequently appear in groups with the same sign, yet they become here so great—up to 0.04 Å.—that they cannot possibly be explained by the systematic difference between Kayser's measures and mine, which was mentioned in my former paper. Unfortunately, in the case of these normals of the third order, the tendency is much more common for different observers to obtain different values for the wave-lengths, in spite of similar methods of observation and in spite of the most careful reference to the normal lines.

Exner and Hascheck[1] find similar results in working up their wealth of wave-length material, and enter into full particulars about it, citing also the work of other investigators on this subject. The differences between my values and those of Kayser are given in the table under the heading "λ−K." The systematic differences mentioned above are naturally still contained in these values and cannot at present be separated from them. They will hardly reach a maximum value of 0.01 Å.

As to the agreement of the measures among themselves, on account of the many broad, hazy, and weakened lines in the yellow-red part of the spectrum, the mean error of the determination of one line is, on the average, from the whole series, somewhat greater, namely $\epsilon = \pm 0.0027$ Å. (167 lines, each in general meas-

[1] *Die Spektren der Elemente bei normalem Druck*, 1, 27, 1911.

ured about seven times) as compared with $\epsilon = \pm 0.0022$ Å. of the earlier investigation.

In order to investigate the cause of the great difference between Kaysers measures and mine, I also determined the wave-lengths of certain lines by means of a Fabry-Perot interferometer. The apparatus itself is sufficiently well known, since Fabry and Buisson, and Eversheim and Pfund have used it in their determination of the normals of the second order. The distance between the half-silvered plates in my investigation amounted to 5 mm, and since it could be referred directly to the normals of the second order a determination of the phase displacement was not necessary.

The following table contains some measures of wave-lengths which belong to the region covered by my previous paper. Under "G_1" the values are the means of three interference rings upon a plate. The column headed "G" gives the values which I obtained with the plane grating. As may be seen the results obtained

G	G_1	G	G_1
I.Å.	I.Å.	I.Å.	I.Å.
4282.408	.407	4422.574	.574
94.130	.128	27.314	.315
4315.089	.085	30.623	.626
37.052	.052	42.349	.345
52.741	.741	59.127	.124
69.778	.777	61.660	.657
75.934	.935	66.555	.557
		94.573	.572

by the two methods agree to within a few thousandths of an Ångström.

Upon further interference investigations in the region from λ 5300 to λ 5500, in which there are great discrepancies between Kayser's values and mine, I found that many of the critical lines showed either very weak or absolutely no interference phenomena, and that only a part of the lines could be measured in this way. In all of the photographs which I had made up to this time either with the plane grating or with the interference apparatus, I had used a relatively short arc, about 3 to 4 mm, and a screen with a 2 mm opening had been placed in front of the slit to cut off the glowing electrodes, so that only the middle part of this short arc was effective.

It occurred to me to change the manner of producing the arc in order to get a sharper interference, and a decided improvement appeared when, with the same strength of current, the arc was made larger, and again only the middle part of about 2 mm of this long arc used. It is evident also that by changing the length of the arc, the structure of many of the iron lines is also changed, and that with the shorter arc the light of the lines in question is less effective for producing interference, and the line also becomes broader. Whether we shall find a displacemnt of the line or not depends upon the nature of the broadening, whether it is symmetrical or unsymmetrical.

I have now measured a small number of lines with the interferometer with an arc about 10 mm long, of which only the middle part was used. The corresponding plates with a short arc could not be measured, because the interference phenomena either were too faint or were wholly lacking. I had for comparison only my own measures, made with the plane grating and an arc about 3 mm long. The results are collected in the following table. Kayser's values and the international normals of the second order are also given.

Goos, Plane Grating 3 mm Arc	Kayser	Goos, Interference 10 mm Arc	International Normals
I.Å.			
5324.195	.197	.196	.196
41.035	.027	.034	
64.851	.862	.870	
71.495	.490	.498	.495
5410.878	.902	.915	
15.170	.184	.203	
24.033	.050	.066	
34.526	.527	.529	.527
97.522	.521	.518	.522

The lines 5365, 5410, 5415, and 5424 are in Kayser's list somewhat strongly displaced toward the red with the 10 mm arc as compared with the values obtained with the 3 mm arc. It is probably safe to assume that Kayser's plates were made with an arc whose length was somewhere between 3 and 10 mm. It may be mentioned that the international normals of the second order apparently show no variability, but we cannot, for this reason,

definitely consider them as constant, for, in every case, these normals have been used as reference lines, and any uniform displacement of these four lines would not appear. Nevertheless a great variability is not probable since the appearance of their interference rings hardly changes with the change in length of the arc. And, even with a very short arc, the interference is easily visible. The normals of the second order are, as we know, especially fine, sharp lines easily capable of producing the interference phenomena. It is of great practical significance that, in this part of the spectrum, the intensity of the arc diminishes rapidly with the length. The middle part of a 10 mm arc requires from 10 to 20 fold longer exposure time than the 3 mm arc. A further confirmation of the displacement of the lines is furnished by a plate which I was able to make with Kayser's concave grating, while on a visit in Bonn. I used the spectrum of the second order, in which 1 Å. = 1 mm, and made two exposures of the region about λ 5600. The first was made with a short arc, 3 to 4 mm; the second with an arc from 8 to 9 mm. In both cases the strength of current remained the same, between 7 and 8 amperes, and a screen was placed in front of the slit so that each time only 2 mm of the middle of the arc was used. The following table contains the results from the above-mentioned plate, and also Kayser's values, my plane grating measures, and the international normals of the second order which were used as reference lines.

Goos, Plane Grating 3 mm Arc	Goos, Concave Grating 3 to 4 mm Arc	Kayser	Goos, Concave Grating 8 to 9 mm Arc	International Normal
I.Å.				
5554.862	.872	.878	.893	....
63.614	.604	.608	.608	....
65.672	.689	.685	.704	....
69.633	.632	.630	.632	.633
72.854	.852	.858	.856	....
76.105	.100	.102	.104	....
86.770	.773	.774	.772	.772
98.270	.288	.292	.307	....
5602.958	.961	.962	.964	....
15.663	.659	.667	.660	.661
24.553	.559	.562	.558	....
38.276	.279	.289	.272	....
58.835	.837	.846	.836	.836

Here again there is a marked variability with the change in length of the arc especially conspicuous in the case of the lines 5555, 5566, 5598. Similar line displacements may naturally be expected if the radiation is investigated away from the middle of the arc, in the immediate vicinity of the glowing electrodes, or if the strength of the current and voltage are changed, or if iron rods of different sizes are used, etc. Fabry and Buisson[1] have recently published a thorough investigation of the great differences in appearance of the different parts of the arc in the various regions of the spectrum. From their paper may be derived many clues leading to the investigation of this question perhaps the most important of all the questions involved in the determination of normal wave-lengths.

From the results thus far obtained it seems to me to be conclusively shown that, for the establishment of a normal system of wave-lengths an exact definition of the light-source is absolutely necessary. In the program of the International Union for Solar Research[2] it is merely specified that the arc shall burn with a strength of current from 5 to 10 amperes, but that is not enough, at least not in the spectral region from λ 5000 to λ 6500.

It is to be hoped that, in the case of the normals of the second order, the lines have been selected with such care that they will not be affected by any variability in the nature or length of the arc. Henceforth, however, we must assume that the great differences in the wave-length determinations of the three observers who have used interference methods, and the differences between their values and those obtained by gratings must be reconsidered, and, in the case of the lines under consideration, new determinations of their wave-lengths should be made, after an exact definition of the light source has been made.

This brings us to the question of the definition of the light-source and indeed as to whether it is possible to find a simple definition. In the yellow-red part of the spectrum, which is the most difficult for exact measurement, one may obtain relatively sharp lines, by using the middle part of a long arc, but the intensity

[1] *Journal de physique* (4), **9**, 929, 1910.

[2] *Trans. Intern. Union for Solar Research*, **1**, 238, 1906.

λ	Intensity	No. of Plates	Mean Error	International Normal	λ−I.N.	λ−K	Remarks
I.Å.			I.Å.		I.Å.	I.Å.	
5328.040	4	7	±0.001			0.000	
28.535	3	7	3			+ 3	
32.912	2	7	2			+ 8	
39.949	3	7	1			+ 2	
41.035	3	7	3			+ 8	*Fe, Mn*
53.390	1	6	4			0	
64.851	3 *u*	7	3			− 11	
65.404	1	6	3			0	
71.495	4	9	1	.495	0.000	+ 5	
79.588	1	5	3			+ 5	
83.353	4	8	2			− 7	
89.476	2	3	6			− 6	
93.186	2	8	2			0	
97.133	4	8	2				
5400.504	2 *ur*	8	3			− 6	
05.780	4	9	1	.780	0	+ 2	
10.878	3	8	2			− 24	
15.170	4 *u*	8	4			− 14	
24.033	4 *u*	8	3			− 17	
29.700	4	8	1			− 1	
34.526	3	8	1	.527	− 1	− 1	
45.024	2	6	2			− 13	
46.924	3	6	3			+ 6	
62.952	2 *u*	5	6			− 12	
63.253	3 *u*	6	3			− 19	
66.430	1	3	4			+ 6	
73.910	2	6	6			0	
76.300	1	5	2			+ 6	
76.586	2	4	2			+ 5	
97.522	3	9	1	.522	0	+ 1	
5501.473	3	8	1			+ 4	
06.784	3	8	1	.784	0	+ 1	
22.467	1	5	5				
25.558	2	6	4			− 1	
35.419	2	7	1	.418*	+ 1	− 1	
43.178	2	7	3			− 6	
43.945	2	7	5			+ 1	
54.862	3	7	2			− 16	
63.614	3	7	2			+ 6	
65.672	3	7	4			− 13	
69.633	5	8	1	.633	0	+ 3	
72.854	6	7	2			− 4	
76.105	4	7	3			+ 3	
86.770	7	7	1	.772	− 2	− 4	
98.270	3	7	3			− 22	
5602.958	4 *uv*	7	2			− 4	
15.663	8	8	1	.661	+ 2	− 4	
24.553	4	7	4			− 9	
33.973	2	7	3				
38.276	3	7	4			− 13	
41.445	2	7	6				
49.696	1	6	5				*Ni*
58.835	5 *uv*	8	1	.836	− 1	− 11	
62.528	4	6	1			− 9	

λ	Intensity	No. of Plates	Mean Error	International Normal	λ−I.N.	λ−K	Remarks
I.Å.			I.Å.		I.Å.	I.Å.	
5678.983	2	6	±0.003			−0.042	*Ni*, Kayser gives intensity 1 *u*
82.180	3	6	4				
91.499	1	6	5				
93.618	1	6	2				
5701.543	3	6	3			− 18	
15.082	3	6	4			− 29	*Ni*
17.839	2	6	4			− 24	
31.759	2	6	5			− 15	
41.859	1	6	7				
48.346	1	4	4			− 35	*Ni*
53.136	3	6	2			− 6	
54.660	3	6	3			− 27	*Ni*
60.850	2	8	2	.843*	+0.007		*Ni*
63.010	4	8	1	.013	− 3	− 4	
75.103	2	6	4				
82.142	3	6	1			− 14	*Cu*, close pair Fabry and Perot 5782.090, 5782.159. Kayser gives intensity 1
91.059	2	6	5			+ 15	
98.204	1	5	4				*Fe*, *Ni*
5805.204	2 *u*	7	3	.211*	− 7	− 8	*Ni*
06.722	1 *u*	6	6				
09.260	1	6	2				
16.342	3	6	2				
31.630	2 *u*	6	7				*Ni*
47.006	1	6	3			+ 10	*Ni*
56.070	1	6	4				
57.761	3	7	1	.760*	+ 1	+ 8	*Ni*
59.633	3 *u*	7	4				
62.338	4	7	4			+ 16	
83.850	3	7	6			+ 28	
92.888	4	7	2	.882*	+ 6	− 9	*Ni*
5905.694	2 *ur*	6	5			+ 33	
14.144	4 *u*	6	3			+ 2	
16.237	1	6	2			− 4	
30.156	4	6	3			+ 4	
34.673	3	7	3	.683*	− 10	+ 5	
52.744	2	7	2	.739*	+ 5	+ 5	
56.692	1	5	4			− 3	
75.352	2	6	5			− 2	
76.796	2	6	5			− 4	
83.703	3 *uv*	6	5			− 6	
84.788	3	6	2			− 7	
87.040	3	6	5			− 5	
6003.041	3	7	1	.039*	+ 2	+ 12	
07.956	1 *u*	5	5			+ 3	Kayser gives intensity 3
08.587	3	7	2			+ 7	
13.519	3	7	2			+ 3	*Mn*
16.674	3	7	2			+ 8	*Mn*
20.139	3 *uv*	7	3			− 28	

λ	Intensity	No. of Plates	Mean Error	International Normal	λ−I.N.	λ−K	Remarks
I.Å.			I.Å.		I.Å.	I.Å.	
6021.829	3	7	±0.001			+0.008	*Mn*
24.051	4	7	3			+ 4	
27.057	3	8	1	.059	−0.002	− 5	
42.084	1	7	1			− 8	
55.980	2	7	4			− 12	
65.492	4	8	1	.492	0	+ 3	
78.466	2	7	2			− 10	
89.570	1	3	7			− 11	
6102.177	2 *u*	7	3			− 5	
03.167	1 *u*	7	3			− 25	
27.919	1	6	3			− 8	
36.622	5	7	1			− 6	
37.000	1	6	4			− 12	
37.701	5	9	1	.701	0	− 2	
41.760	2	8	3				
47.846	1	8	5			+ 2	
51.633	1	7	4			0	
57.736	2	8	2			− 4	Kayser gives intensity 4
65.370	1	8	3			− 17	
73.347	2	8	2			− 3	
80.218	1	7	2			− 12	Kayser gives intensity 3
88.089	1	7	3				
91.569	5	9	1	.568	+ 1	+ 1	
6200.324	2	8	3			− 6	Kayser gives intensity 5
13.447	3	8	2			+ 9	
15.152	1	8	4			− 1	
19.294	3	8	1			+ 5	
27.266	1	8	5				
30.732	5	9	1	.734	− 2	− 2	
32.673	3	8	1			+ 4	
40.662	1	8	4			− 4	
46.351	5	8	2			+ 6	
52.566	5	8	2			− 1	
54.271	3	8	2			+ 2	
56.279	2	8	2			+ 2	
65.146	3	9	1	.145	+ 1	+ 1	
70.238	2	8	2			− 7	
80.626	2	8	4			− 5	
90.956	2 *u*	8	3				
97.803	3	8	2			− 1	
6301.531	5	8	1			+ 3	
02.525	3	8	1			+ 3	
15.322	2	8	1			− 1	
15.803	1	8	4			− 19	
18.028	4	9	1	.028	0	− 3	
22.697	3	8	1			− 1	Misprint in Kayser
35.341	4	8	1	.341	0	+ 2	
36.855	5	7	2			+ 5	
44.165	2	7	1			+ 1	
55.049	3	7	2			+ 4	
58.675	2	7	2			− 18	
62.882	1	7	4				
80.753	2	7	1			− 1	

λ	Intensity	No. of Plates	Mean Error	International Normal	λ−I.N.	λ−K	Remarks
I.Å.			I.Å.		I.Å.	I.Å.	
6393.611	6	9	±0.001	.612	−0.001	−0.001	
6400.036	8	8	2			0	
08.058	4	8	2			+ 13	
11.685	6	8	1			+ 12	Kayser gives intensity 4
21.366	5	8	2			+ 12	Kayser gives
30.860	5	8	1	.859	+ 1	+ 12	intensity 3
62.743	3	6	2			+ 9	
69.237	2 *u*	5	3				
75.642	1	6	2				
81.885	2	6	3			− 11	
91.372	2	6	3				
94.992	7	6	1	.993	− 1	− 2	

of the light there is so weak that the necessary exposure time would be altogether too long. On the other hand, the short arc and the region near the negative pole have the advantage that the light which they emit is extremely intense, against which is the disadvantage that the lines which they produce are broader and weaker and poorly adapted to interference observations. The advantage of greater intensity, in this photographically inactive part of the spectrum, seems to me more important than the sharpness of the lines. On the other hand, one should avoid the vicinity of the negative pole for the blue and violet on account of the many reversals. These, to be sure, admit of very accurate settings in their measurement, but the reversal is often unsymmetrically situated in the emission line. As far as my observations extend into the violet, down to λ 4000, the wave-lengths are approximately constant. The few lines which are doubtful could easily be omitted since there are plenty of other lines left, in contrast to the red part of the spectrum, where the lines in some places are few and rather poor. In some cases, if a long slit is to be used, as for example, in connection with a large concave grating, a long arc will be preferred, so that the slit may be well illuminated. If a short arc is used, it will be necessary to enlarge it by means of a condensing lens. It might also be advantageous to try a weaker current of perhaps 2 or 3 amperes, and smaller iron rods of 4 or 5 mm diameter. Moreover, the insertion of a piece of ground glass in front

of the slit might be considered, so that all parts of the arc might be effective, but then, in the case of great dispersion, or in the red region, the exposure time would be much too long. Thus it seems to me that it will be very difficult to find a perfectly satisfactory definition of the light source. It should also be emphasized that the nickel arc is not a satisfactory substitute in the region between λ 5600 and λ 5900, where there are so few iron lines. The nickel lines are nearly all hazy and weakened and show displacements up to several hundredths of an Ångström.

At the present time, therefore, there seems to be no prospect of the establishment in the near future of a normal system of wave-lengths, accurate to a few thousandths of an Ångström over the whole spectrum. Further investigation into the phenomena of the electric arc is what is most needed now in this realm of research.

HAMBURG, PHYSIKALISCHES STAATSLABORATORIUM
July 1912

SPECTRA OF LOW POTENTIAL DISCHARGES IN AIR AND HYDROGEN

By GORDON S. FULCHER

The light emitted as a direct result of the bombardment of nitrogen, hydrogen, and oxygen molecules by cathode rays has been investigated in the case of rays with velocities corresponding to from 500 to 5000 volts difference of potential.[1] An attempt has since been made to determine the effect of slower rays, with the following results.

In the first apparatus made, a Wehnelt cathode was placed parallel to a perforated anode at a fixed distance of 1 mm. By applying a field, the electrons set free at the cathode could be projected through the hole in the anode so as to form a beam of cathode rays of a definite velocity in the chamber behind the anode. However, no visible beam was obtained except when conditions were such as to permit a discharge to pass directly between the electrodes. Evidently ionization by collision was necessary to produce a sufficient number of rays. The apparatus was therefore changed so as to permit a variation of the distance between the electrodes (see Fig. 1). It was then discovered that when they were about 5 mm apart, with a pressure of the order of 0.1 mm *Hg*, a discharge could be obtained with a very low difference of potential by using a sufficiently hot cathode. The minimum observed for hydrogen was 20 volts, for air 27 volts, for oxygen 45 volts, with a discharge current of about 0.04 ampere. The gases were dried by passing over phosphorus pentoxide, and the impurities given off by the hot cathode were washed away by maintaining a continuous stream of gas through the tube with the aid of a capillary and a Gaede pump; but otherwise no special precautions were taken to purify the gases. The spectrograms show slight traces of mercury lines, and the *CO* bands were present when oxygen was used. These minimum potentials were best obtained just after inserting a new, salted, platinum cathode, but

[1] Fulcher, *Astrophysical Journal*, **34**, 388, 1911.

could not be maintained for more than half an hour without renewing the cathode, as the discharge potential always rose steadily while the discharge was running.

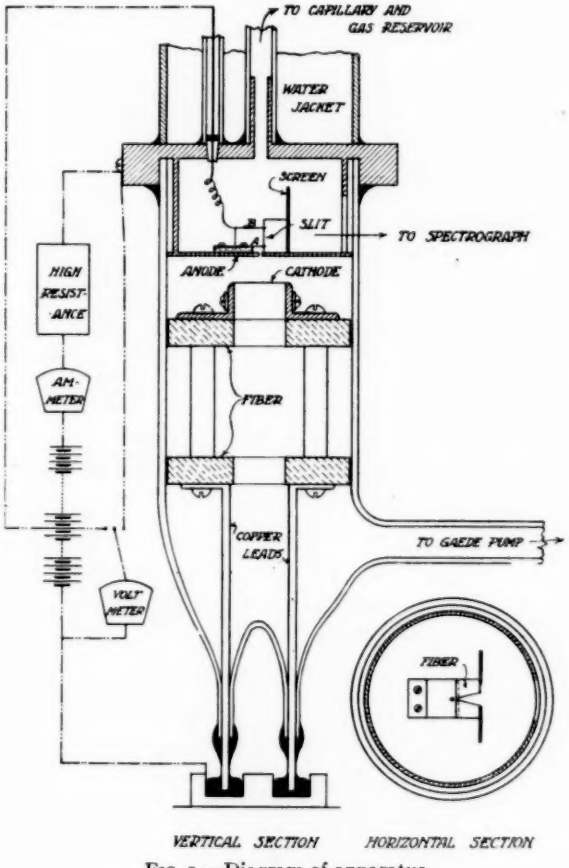


FIG. 1.—Diagram of apparatus

AIR

In the case of air, at the higher voltages (50 volts), distinct bluish columns could be seen rising from various points on the salted cathode to the anode through the reddish glow which filled the discharge chamber. Their bases were distinctly separated from the cathode by a short dark space (0.5 mm). As the tem-

perature of the cathode was increased, suddenly the discharge potential dropped, the bluish columns vanished and left only the reddish glow, separated, however, by a wider dark space from the cathode. Photographs recording the changes in the spectrum accompanying the decrease in the discharge potential were made with a Hilger constant deviation spectrograph. Three are shown enlarged four times in Fig. 2, Plate II. Note the striking decrease in the relative intensity of the negative nitrogen bands with respect to the positive bands. Under the most favorable circumstances they disappear entirely, as is shown by No. 193. It was found important to analyze only the light produced between the two electrodes, as the discharge potential is no measure of the potential differences which may exist between the cathode and other parts of the tube; at least that is the probable explanation of the fact that spectrograms of the light from the region behind the cathode showed the negative bands faintly even when the discharge potential was less than 30 volts. Cramer "spectrum" plates were used, but no variations in the relative intensity of the first (red) and second groups of nitrogen positive bands were observed.

Now to return to the problem of the production of light by slow cathode rays. The light produced by the rays projected through the perforated anode proved too faint for spectroscopic analysis; so a pair of plates, A and B (Fig. 1), were placed behind the anode and insulated from it in order that a retarding potential might be applied and a larger number of slow rays might be obtained than if the potential of the discharge were no greater than that corresponding to the desired low velocity of the rays. A slit with fiber jaws was placed close to the beam. However, it was found difficult to get and maintain a beam sufficiently bright for spectrographic analysis. The most interesting spectrogram obtained is shown in Fig. 2, No. 202. The positions of Plates A and B are indicated. It is clearly seen that as the rays are retarded, the intensity of the negative nitrogen bands decreases very greatly, while the positive bands become stronger, not only relatively but absolutely, in spite of the dispersion of the rays, until, halfway between A and B, the positive band λ 4269 is more intense than the negative band λ 4278. During the exposure the plates were

PLATE II

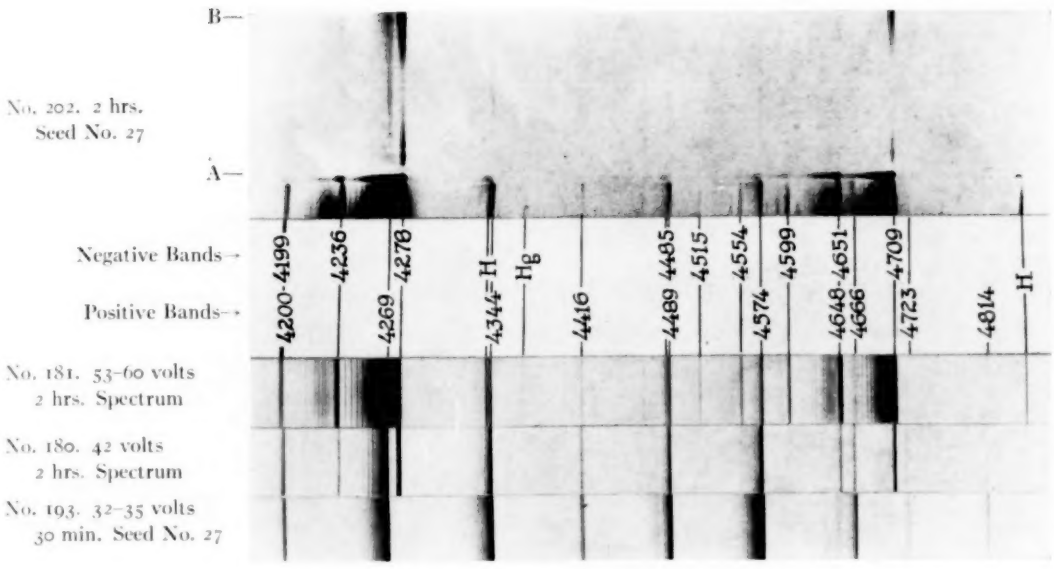


FIG. 2.—Air

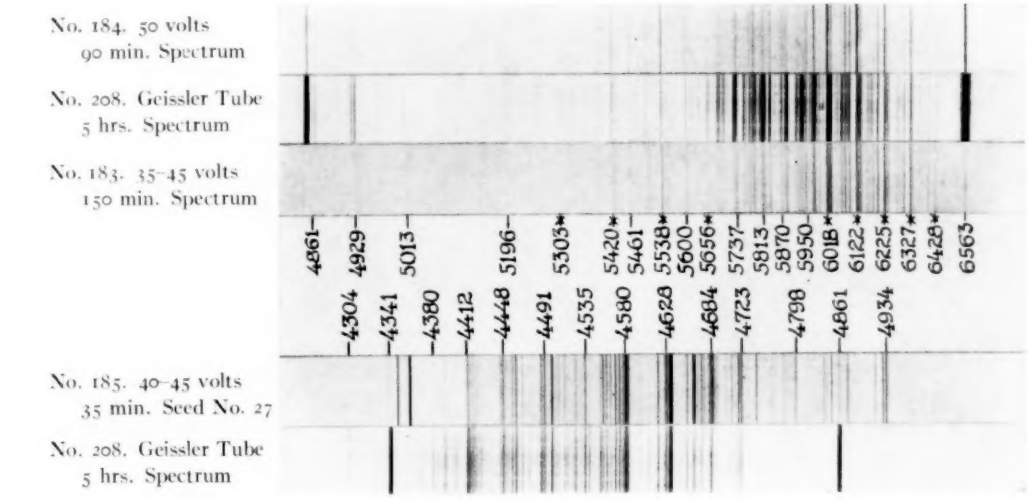


FIG. 3.—*H*

maintained at a potential 20 volts above that of the cathode, and the discharge potential was about 80 volts. However, it may well be that because of the presence of large numbers of free electrons and ions the lines of force passed through the hole in Plate A and the retarding potential was not as great as the above figures would indicate; so that the spectrum of light produced halfway between A and B may correspond to a greater energy of the rays than would be produced by a fall through 20 volts. To decrease the difference between the actual and the measured retarding potential differences, the apparatus was modified as shown in Fig. 4. The two brass

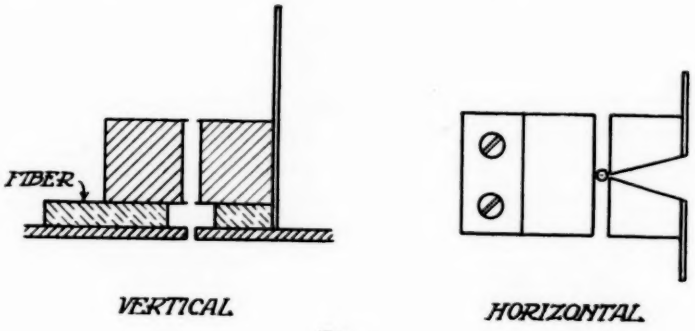


FIG. 4

jaws of the slit, together with a brass block, formed a narrow chamber with parallel walls 1 mm apart into which the rays passed through a small round hole. However, the light-intensity was reduced to such an extent that no satisfactory spectrogram was obtained.

The light analyzed in this experiment came from the path of the cathode rays; hence it was chiefly due directly to the impact of the cathode rays on the gas molecules. At the pressures used, the mean free path was several millimeters; hence, in comparison with the ionization produced directly by the cathode rays, any secondary reactions, such as the recombination of ions formed by the rays, would be scattered through a larger volume around the beam, a smaller proportion of the light produced would pass through the spectrograph, and the effect would not be localized but would be more uniformly distributed along the slit. Therefore we may

conclude that the variations in the spectrum obtained are due to the variations in the energy of the cathode rays; that the light is emitted as a direct result of the ionization produced by the rays; that slow cathode rays may excite the nitrogen molecules ionized by them to emit both positive and negative band spectra, whose relative intensity depends on the energy of the ionizing rays. But we found above that the spectrum of the light from between the electrodes in the discharge chamber showed variations with discharge potential similar to the variations shown by the spectrum produced by the cathode rays. It seems probable, then, that the light in the discharge chamber at these low voltages is chiefly the direct result of ionization by cathode rays; that the slowest rays (30 volts) excite the emission of only the positive nitrogen bands; but that the relative intensity of the negative bands increases very rapidly with the energy of the exciting rays so that for 50 volts they are the stronger and for higher discharge potentials greatly exceed in intensity the positive bands. Both series of bands seem to be emitted by the nitrogen molecules when positively ionized by cathode rays. Perhaps the faster rays can penetrate farther into the molecules and arouse a simpler and more fundamental type of vibration than the slower rays are able to do; or the positive bands may be emitted by molecular aggregates dissociated by the faster rays.

Gehrcke and Seeliger[1] recently described a very pretty experiment in which they shot a cathode beam from a Wehnelt cathode obliquely into a retarding electric field sufficiently strong to cause the rays to describe a parabolic path. Under those conditions they observed marked differences in color between different parts of the beam. In air the beam changed from bluish to reddish at a point where the energy of the rays corresponded to about 46 volts, and became non-luminous where the energy corresponded to less than 29 volts. The above results show that the change from bluish to reddish means a decrease in the relative intensity of the negative nitrogen bands, but that the change is not discontinuous but takes place continuously, though rapidly, between 50 and 30 volts. Their value for the minimum energy necessary to produce

[1] *Verhandlungen der Deutschen physikalischen Gesellschaft*, **14**, 335, 1912.

luminescence agrees well with the minimum observed discharge potential reported above, 27 volts. The changes in hydrogen described below also agree with their results except that the minimum observed potential was 20 volts instead of 29 volts, the value reported by them. The lower value may be due to the presence of a trace of mercury vapor in my discharge tube, as they found that the minimum energy for mercury vapor was only 10 volts. Yet spectrograms taken when the discharge potential did not rise above 26 volts show the mercury lines no stronger than the main compound lines of hydrogen.

The relations found between the nitrogen band spectra and the energy of the exciting rays should prove useful in the investigation of phenomena in the positive column and other parts of a discharge tube. According to Sir J. J. Thomson's[1] theory of striations, there are regularly spaced, self-perpetuating variations in the density of free electrons corresponding to the variations in luminosity observed; the distance apart of the striations is so adjusted that the electrons set free in one striation, and accelerated by the field due to the distribution of free electrons present, acquire before reaching the next just sufficient energy to cause ionization and luminescence. The positive column spectrum in air is known to consist chiefly of the positive nitrogen bands. Perhaps if the faster cathode rays from the cathode and other parts of the tube were excluded, the negative bands would vanish and we should know that we were dealing merely with cathode rays having velocities corresponding to a fall through about 30 volts, and thus confirm Thomson's theory. As soon as we know which electrical molecular changes result in the emission of each distinct spectrum associated with an element, we shall have a powerful tool for the solution of the remaining problems relating to the discharge of electricity through gases.

HYDROGEN

The changes in the appearance of the discharge in hydrogen as the discharge potential decreased were so similar to those observed in air that no further description need be given. Some

[1] *Phil. Mag.*, **18**, 441, 1909.

of the spectrograms of the light from between the two electrodes are shown enlarged four times in Fig. 3, Plate II. Between 60 volts and 40 volts the main series lines decreased greatly in intensity with reference to the lines of the compound spectrum. The ratio of the intensities of the series lines on plates Nos. 208 and 183 is several hundred as determined by taking short exposures with the Geissler tube, while the lines λ 6327, λ 6225, λ 5538, etc., are equally intense on both. The relative intensity of the two spectra did not change appreciably when the discharge potential was decreased from 40 to 23 volts. On no spectrogram obtained are the series lines entirely absent. Perhaps water is more easily dissociated into hydrogen atoms by cathode rays than hydrogen molecules are, and the residual series lines may be due to a trace of water present; or the weak lines may actually belong to the compound spectrum, a possibility suggested by the fact that their relative intensity with respect to the compound lines was found to be about the same on several spectrograms obtained under varying conditions.

There are also very interesting changes in the compound line spectrum itself to be observed by comparing No. 183 with No. 208. The spectrum for the low voltages is evidently simpler; the strongest lines seem to be arranged with some regularity, suggesting bands. In regard to the existence of bands in the compound spectrum, Dufour[1] states that some of the weakest lines have equal frequency differences, an arrangement resembling that of bands, so that the compound spectrum consists of a complicated line spectrum superposed on a faint band spectrum; but he gives no numerical data. Otherwise there seems to have been no mention of the existence of numerical regularities in the second spectrum of hydrogen. A year ago in glancing over Kayser's reprint of Watson's measurements,[2] I was struck with the recurrence of strong lines in the red at intervals of about 100 Å, and an investigation showed that the differences in wave-length between certain strong lines in the red and yellow are so regular as to prove that they together form groups resembling bands. It was very interesting to find later that these "band" lines are the strongest lines

[1] *Annal. chim. phys.* (8), **9**, 416, 1906.

[2] Kayser, *Handbuch der Spectroscopie*, V. Band, 494–502, 1910.

in the low-potential discharge spectrum of hydrogen. Some of the lines were investigated by Dufour[1] and none were found to show a Zeeman effect.

The following table contains the "band" lines and the differences in wave-length between successive members of the series.

TABLE I

MAIN BAND LINES OF HYDROGEN

	Å	Difference	Å	Difference	Å	Difference	
S_1.....	.....	.....	6441.83(0)	101.08	6340.75(2)	102.18	
S_2.....	.....	.....	6433.80(0)	101.10	6332.70(1)	102.22	
S_3.....	6527.63(1)	99.27	6428.36(2)	101.11	6327.25(8)	102.23	
S_1–S_3....	.....		13.47		13.50		
S_2–S_3....	.....		5.44		5.45		
S_1.....	6238.57(6)	103.01	6135.56(6)	103.42	6032.14(5)		Head
S_2.....	6230.48(2)	102.95	6127.53(2)	103.53	6024.00(4)		Head
S_3.....	6225.02(10)	103.01	6122.01(10)	103.48	6018.53(9)		Head
S_1–S_3....	13.55		13.55		13.61		
S_2–S_3....	5.46		5.52		5.47		
S_1.....	5671.10(0)	118.50	5552.60(0)	117.54	5435.06(1)	117.00	
S_2.....	5661.97(0)	118.26	5543.71(0)	117.51	5426.20(1)	117.10	
S_3.....	5655.98(1)	118.31	5537.67(5)	117.57	5420.10(4)	116.75	
S_1–S_3....	15.12		14.93		14.96		
S_2–S_3....	5.99		6.04		6.10		
S_1.....	5318.06(0)	.....	.....	.....	.....	.....	
S_2.....	5309.10(0)	.....	.....	.....	.....	.....	
S_3.....	5303.35(2)	.....	.....	.....	.....	.....	
S_1–S_3....	14.96		.....		.....		
S_2–S_3....	5.75		.....		.....		

Mean S_1–S_3.....	13.54 (first band)	15.00 (second band)
Mean S_2–S_3.....	5.47 (first band)	6.04 (second band)
Ratio.....	0.404 (first band)	0.403 (second band)

The wave-lengths are those given by Watson;[2] the agreement between the differences of S_1, S_2, and S_3 shows the accuracy of his measurements. The wave-lengths determined by Hasselberg[3]

[1] *Op. cit.*, pp. 413–416.

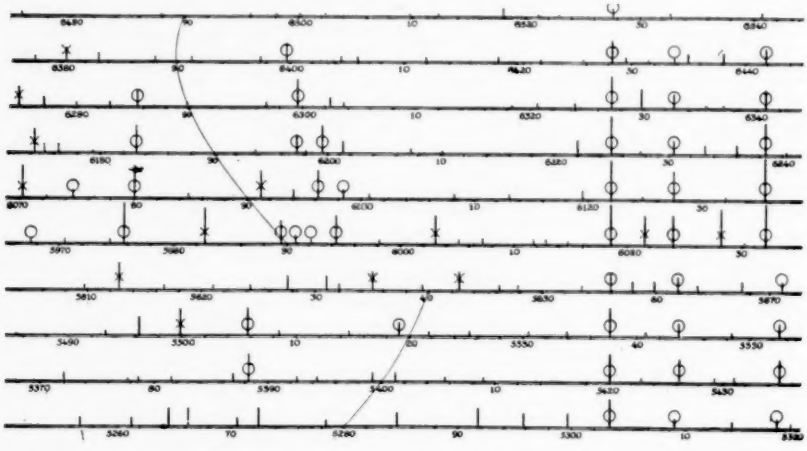
[2] *Proc. Roy. Soc.*, A, **82**, 189–204, 1909.

[3] *Mém. acad. St. Petersb.* (7), **31**, No. 14, 1883. Reprinted in Kayser's *Handbuch*, and in Dufour's paper in *Annal. chim. phys.* (*loc. cit.*).

show much greater variations. The intensities given by Watson are added in brackets.

The two bands seem to be related since the ratio of S_2–S_3 to S_1–S_3 is the same for both. There are other lines which seem to belong with these groups or bands:

S_4—λλ 6303.68(0), 6201.38(2), 6098.45(3), 5994.30(4);
S_5—λλ 6199.58(4), 6096.21(4), 5992.14(0);
S_6—λλ 6299.60(5), 6197.32(2), 6094.03(1), 5989.47(3);
S_7—λλ 6285.56(4), 6183.19(4), 6080.03(9), 5975.68(8).



Lines of group one. Lines of group two.

FIG. 5

The regularities are shown graphically in Fig. 5 where the relative positions of the band lines and of all others in their neighborhood whose wave-lengths have been determined by Watson,[1] Porlezza,[2] and Norzi[3] are indicated. The heights of the lines show the relative intensities given by Porlezza and Norzi. The two groups referred to below (Table II) are indicated.

To show more in detail the differences in the compound spectrum for high and low discharge potentials and the relation of these variations to the Zeeman-effect observations of Dufour, the

[1] Watson, *loc. cit.*

[2] Porlezza, *Atti accad. Lincei*, **20** (2), 178, 1911.

[3] Porlezza and Norzi, *ibid.*, **20** (1), 822, 1911.

following table is given. Four plates, two taken with high discharge potential and two with low, were measured, and the intensities of the various lines on the four plates were estimated independently. The agreement was in general very close. The averages for each pair are given in the columns headed "Low" and "High."

TABLE II

RELATIVE INTENSITIES OF LINES IN THE HIGH AND LOW POTENTIAL SPECTRA OF HYDROGEN

Line	Low	High	Line	Low	High	Line	Low	High
Group one—			*5989.47	2	3	*Group two*—		
6441.83	½	½	* 75.68	10	10	6380.30	0	1
28.36	3	3	* 50.15	5	5	6275.10	0	3
6399.71	4	3	* 25.09	3	2	6174.28	0	4
62.67	2½	2	* 16.76(?)	1	5	61.81	½	4
40.75	2½	2	09.71	1	1½	†6091.16	0	5
27.25	9	9	5872.12	5	5	†6070.21	4	6
6299.60	6	5	69.47			*(67.96)		
85.56	3	2	* 36.28	7	10	*(6032.14)	7	12
71.55	1	1	* 22.99	5	6	† 28.21		
* 38.57	5	5	*5786.00(?)	7	10	† 03.08	½	4
30.48	1	1	* 75.28	6	8	†5982.79	½	6
* 25.02	11	11	* 37.06	6	9	† 63.70	2	5
*6199.58	7	10	5671.10	1½	2	† 59.98		
* 97.32			61.97(?)	1	4	† 38.87	3	9
* 83.19	5	4	55.98	3	5	† 31.62	3	9
* 35.56	8	11	5600.65	3	3	* 16.76(?)	1	5
* 27.53	3	4	5597.80			†5779.22	½	3
* 22.01	12	12	52.60	3	2	59.76	2	7
*6096.21	6	6	43.71	1½	2	13.59	1	4
* 80.03	7	8	37.67	10	8	03.50	1	8
†(70.21)	4	6	19.12	1	1	5689.00	3	10
* 67.96			05.79	4	4	88.00		
* 41.27	1	1½	5435.06	2	3	84.33	2	10
* 32.14	7	12	26.20	3	4	42.65	0	3
†(28.21)			20.10	7	7	35.05	½	4
* 18.53	16	14	5388.35	4	4	12.77	½	5
*5994.30	2	2½	03.35	4	4	5499.84	1	3

* No Zeeman effect (Dufour). † Show Zeeman effect (Dufour).

The lines fall into two groups, the first containing all those which are equally intense in both spectra, the second those which are relatively very much weaker in the low potential spectrum. To the first group belong all the lines between λ 5700 and λ 6300 which Dufour found do not show a Zeeman effect, and also all the band lines listed above. To the second group belong all the lines which do show a Zeeman effect. The only exceptions seem to be

λ 5916 and λ 5786. For the latter Dufour first reported no transverse Zeeman effect (collimator perpendicular to magnetic field), but later[1] said that it shows a normal longitudinal Zeeman effect—perhaps a misprint, as the statements made are inconsistent. The lines may be double, only one showing the Zeeman effect, but both lines should be tested again. Then the band line λ 5662 seems too weak in the low potential spectrum. In general, however, it seems to be true that the lines in the hydrogen compound spectrum between λ 5300 and λ 6500 which show the Zeeman effect, like the series lines, are much weaker in the low potential spectrum; while the strongest lines which appear in the low potential spectrum do not show the Zeeman effect and from the regularity of their arrangement evidently belong to a band spectrum of hydrogen.

So far we have spoken only of the red and yellow regions of the hydrogen spectrum. Dufour has also investigated the lines in the blue region[2] and has found between λ 4300 and λ 4700 about 30 lines showing no Zeeman effect, an equal number showing a normal effect to a greater or less extent, and a few abnormal lines. As an examination of the spectrograms Nos. 208 and 185 (Fig. 3) will show, there is little change in the relative intensity of the lines in this region when the discharge potential is varied between extreme limits, and the few changes which do occur do not seem related to the Zeeman effect at all. The following lines seem relatively weaker in the low potential spectrogram:

$\lambda\lambda$ 4551.14, *4543.87, †4538.51, *4514.51, †4474.42.

I have searched in vain for any recurring wave-length differences which would group any of the lines together. The results of Dufour serve to emphasize the complexity of this spectrum, but if they are extended may perhaps furnish the clue necessary to unravel it.

In oxygen, with a discharge potential of 75 volts, the spectrum consisted chiefly of the negative bands in the red and yellow, but the spark and series lines were also present. On lowering the potential to 45 volts, the negative bands disappeared and were

[1] Dufour, *Journal de physique* (4), **8**, 258, 1909.

[2] *Ibid.*

replaced by *CO* bands, but the series lines $\lambda\lambda$ 6156 and 4368 were still present.

The nitrogen, hydrogen, and oxygen bands are probably emitted by the various molecules as a result of ionization by cathode rays.

I regret that some points in this paper have not been investigated more thoroughly, but I present these results, such as they are, since other work will prevent me from carrying the investigation further at present.

MADISON, WIS.
October 22, 1912

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